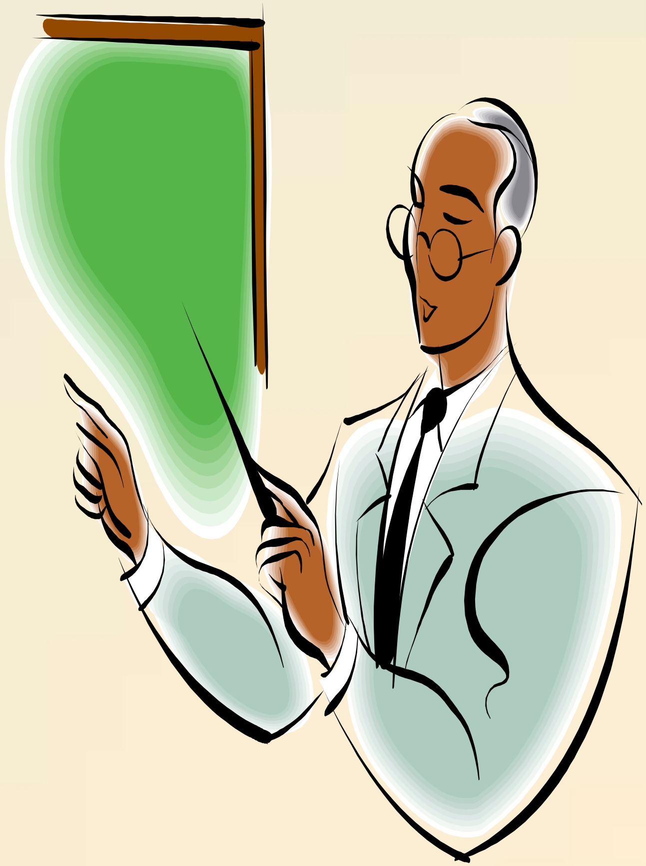
BINOMIAL AND RELATED DISTRIBUTION

RECALL...

- Mean = np
- σ^2 =np(1-p)

  (i.e.) $\sigma = \sqrt{np(1-p)}$



Example :3

Let the random variable Y equal to the no. of success throughout n independent repetitions of a random experiment with probability P of success (i.e.) Y is b(n,p). The ratio Y/n is called the relative frequency of success for every ε>0.

Example :4

Let the independent random variable X_1 X_2 X_3 have the same distribution function $F(x)$. Let Y be the middle value of x_1 x_2 x_3 to determine the distribution function of Y.

Example :6

Consider a sequence of independent random repetition of a random experiment with constant probability P of success. Let the random variable Y denote the number of failures in the sequence before the r-th success. (i.e.) Y+r equal to the number of trials to produce exactly r-success. Here r is a fixed positive integer. To determine the p.d.f of Y.

Definition:

A distribution with p.d.f of the form

$$g(y)=\begin{cases} \binom{y+r-1}{r-1} p^{r} (1-p)^{y} & ,y = 0,1\ldots. \\ 0 & \text{elsewhere} \end{cases}$$

is called a "*negative binomial distribution*" And any such g(y) is called a *negative binomial p.d.f.*

Note :1

The p.d.f of the trinomial distribution is

$$f(x,y)=\begin{cases} \dfrac{n!}{x!y!(n-x-y)} p1^{x}p2^{y}p3^{n-x-y} \\ 0 \text{ elsewhere} \end{cases}$$

where x and y are non-negative integer with x+y<=n and p1p2p3 are positive proper fraction with p1+p2+p3=1.

Note:2

The m.g.f of a trinomial distribution is

$M(t_1,t_2) = (P_1e^{t_1}+P_2e^{t_2}+P_3)^n$ $\forall$ real values of t_1 and t_2.

Note :3

In general, the p.d.f of multinomial distribution is,

$$F(x_1,x_2.....x_n)=\frac{n!}{x_1!x_2!...x_{k-1}!x_k!}p_1^{x1}p_2^{x2}.....p_{k-1}x^{k-1}p_kx^k$$

The m.g.f of a multinomial distribution is

$M(t_1,t_2....t_{k-1})=(p_1e^t+p_2e^t+....+p_{k-1}e^{t-1}+p_k)^n$

Where X_1 X_2...X_{k-1} are random variables.

Definition:

Let X be $b(n,p_1)$ are Y be $b(n,p_2)$. Then the means and variance of X and Y are respective $\mu_1=np_1$, $\mu_2=np_2$ and $\sigma_1^2=np_1(1-p_1)$, $\sigma_2^2=np_2(1-p_2)$. The conditional p.d.f of Y given X=x in binomial distribution is

$$F_{2|1}(y/x) = \begin{cases} \dfrac{(n-x)!}{y!(n-x-y)!}\left(\dfrac{P_2}{1-P_1}\right)^{y}\left(\dfrac{P_3}{1-P_1}\right)^{n-x-y} & y = 0,1,\dots,n-x \\ 0 & \text{elsewhere} \end{cases}$$

Thus the conditional distribution Y given X=x is b[n-x, $P_2/(1-p_1)$.

Hence the conditional mean of Y given X=x is the linear function,

$E(Y/x)=(n-x)(P_2-P_1)$

The conditional distribution of X given Y=y is b[n-y, $P_1/1-P_2$] and thus the conditional mean of X given Y=y is,

$F(X/y)=(n-y)(P_1/1-P_2)$.

Note:

The correlation coefficient of binomial distribution is $P = -\sqrt{(P_1 P_2 / (1-P_1)(1-P_2))}$


THANK
YOU