



BINOMIAL AND RELATED DISTRIBUTION

RECALL...

Mean = np

$\sigma^2 = np(1-p)$

(i.e.) $\sigma = \sqrt{np(1-p)}$



Example :3

Let the random variable Y equal to the no. of success throughout n independent repetitions of a random experiment with probability P of success (i.e.) Y is $b(n,p)$. The ratio Y/n is called the relative frequency of success for every $\varepsilon > 0$.

Example :4

Let the independent random variable X_1 X_2 X_3 have the same distribution function $F(x)$. Let Y be the middle value of x_1 x_2 x_3 to determine the distribution function of Y .

Example :6

Consider a sequence of independent random repetition of a random experiment with constant probability P of success. Let the random variable Y denote the number of failures in the sequence before the r -th success. (i.e.) $Y+r$ equal to the number of trials to produce exactly r -success. Here r is a fixed positive integer. To determine the p.d.f of Y .

A decorative border of colorful 3D numbers (0-9) surrounds the text. The numbers are in various colors like red, yellow, blue, green, purple, and orange, and are arranged in a somewhat chaotic but repeating pattern.

Definition:

A distribution with p.d.f of the form

$$g(y) = \begin{cases} \binom{y+r-1}{r-1} p^r (1-p)^y, & y = 0, 1, \dots \\ 0 & \text{elsewhere} \end{cases}$$

is called a “*negative binomial distribution*”

And any such $g(y)$ is called a *negative binomial p.d.f.*

Note :1

The p.d.f of the trinomial distribution is

$$f(x,y) = \begin{cases} \frac{n!}{x!y!(n-x-y)} p_1^x p_2^y p_3^{n-x-y} \\ 0 \text{ elsewhere} \end{cases}$$

where x and y are non-negative integer with $x+y \leq n$ and $p_1 p_2 p_3$ are positive proper fraction with $p_1 + p_2 + p_3 = 1$.

Note:2

The m.g.f of a trinomial distribution is

$$M(t_1, t_2) = (P_1 e^{t_1} + P_2 e^{t_2} + P_3)^n \quad \forall \text{ real values of } t_1 \text{ and } t_2.$$

Note :3

In general, the p.d.f of multinomial distribution is,

$$F(x_1, x_2, \dots, x_n) = \frac{n!}{x_1! x_2! \dots x_{k-1}! x_k!} p_1^{x_1} p_2^{x_2} \dots p_{k-1}^{x_{k-1}} p_k^{x_k}$$

The m.g.f of a multinomial distribution is

$$M(t_1, t_2, \dots, t_{k-1}) = (p_1 e^{t_1} + p_2 e^{t_2} + \dots + p_{k-1} e^{t_{k-1}} + p_k)^n$$

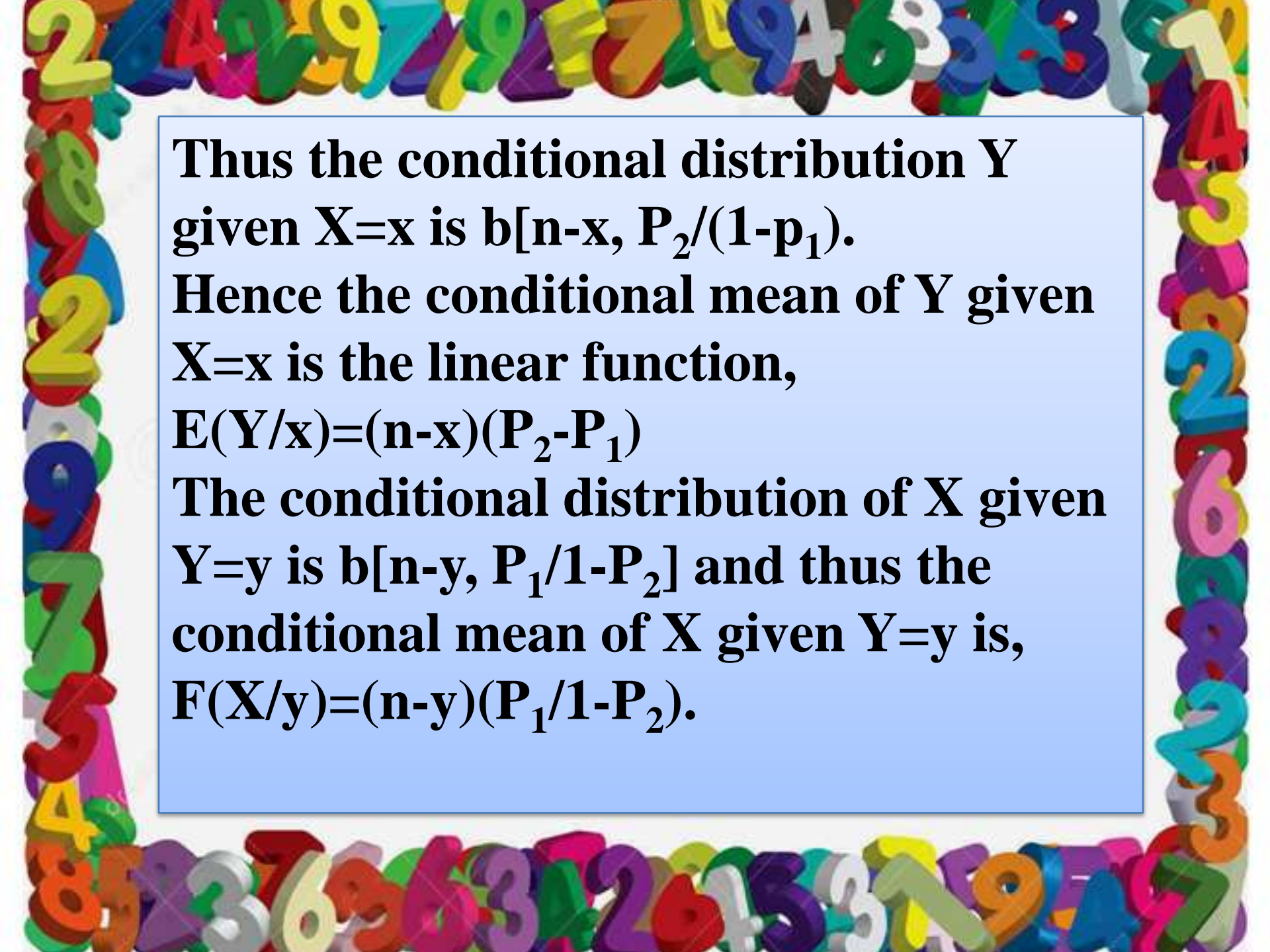
Where X_1, X_2, \dots, X_{k-1} are random variables.

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Definition:

Let X be $b(n, p_1)$ and Y be $b(n, p_2)$. Then the means and variance of X and Y are respectively $\mu_1 = np_1$, $\mu_2 = np_2$ and $\sigma_1^2 = np_1(1-p_1)$, $\sigma_2^2 = np_2(1-p_2)$. The conditional p.d.f of Y given $X=x$ in binomial distribution is

$$F_{2/1}(y/x) = \begin{cases} \frac{(n-x)!}{y!(n-x-y)!} \left(\frac{p_2}{1-p_1} \right)^y & y=0, \dots, n-x \\ \left(\frac{p_3}{1-p_1} \right)^{n-x-y} & y=x+1, \dots, n-x \\ 0 & \text{elsewhere} \end{cases}$$



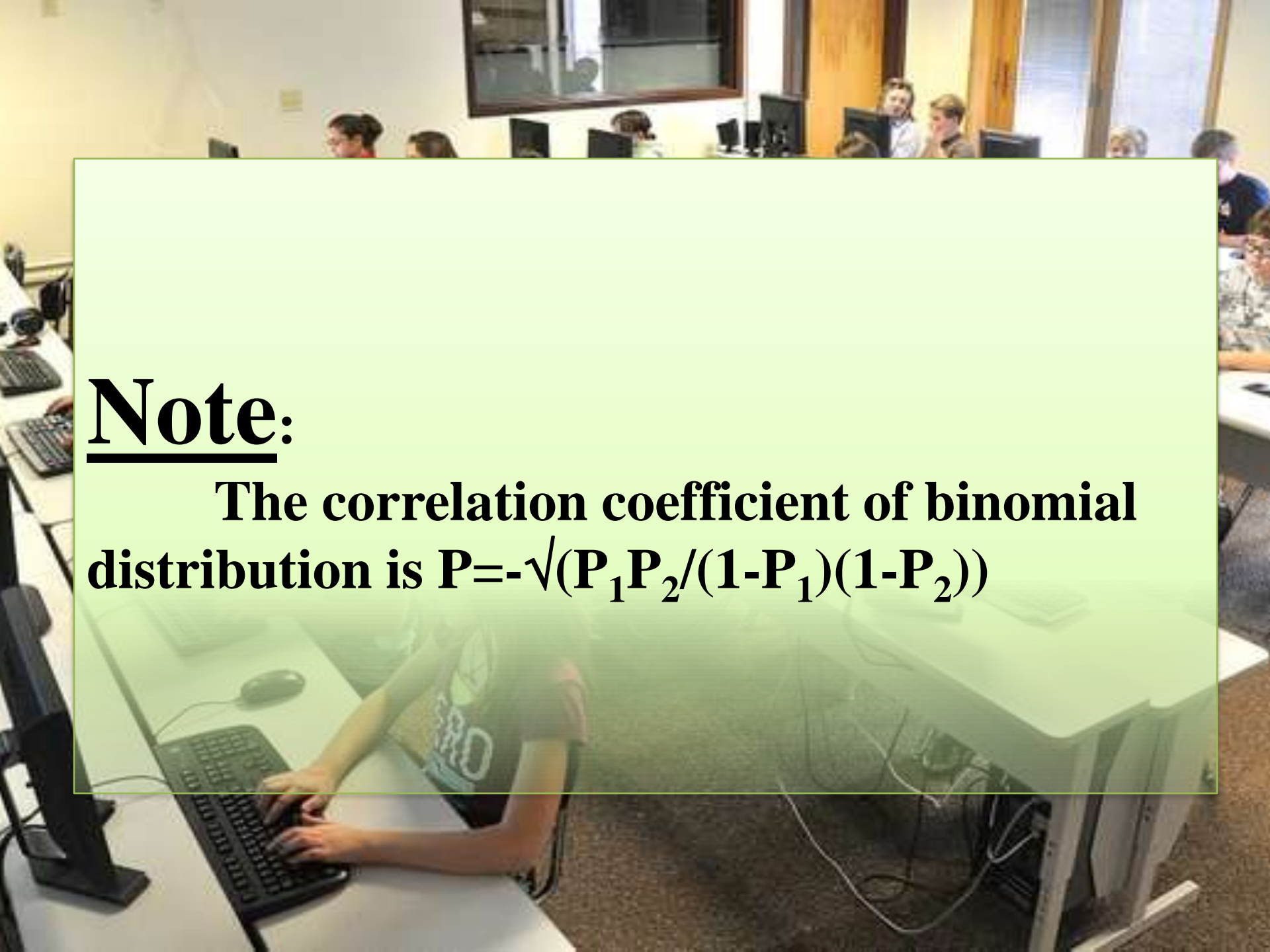
Thus the conditional distribution Y given $X=x$ is $b[n-x, P_2/(1-p_1)$.

Hence the conditional mean of Y given $X=x$ is the linear function,

$$\mathbf{E}(Y/x)=(n-x)(P_2-P_1)$$

The conditional distribution of X given $Y=y$ is $b[n-y, P_1/1-P_2]$ and thus the conditional mean of X given $Y=y$ is,

$$\mathbf{E}(X/y)=(n-y)(P_1/1-P_2).$$

A photograph of a computer lab with several students sitting at desks, working on computers. The room has large windows and a wooden door. A semi-transparent green box is overlaid on the image, containing text.

Note:

The correlation coefficient of binomial distribution is $\rho = -\sqrt{P_1 P_2 / ((1 - P_1)(1 - P_2))}$

