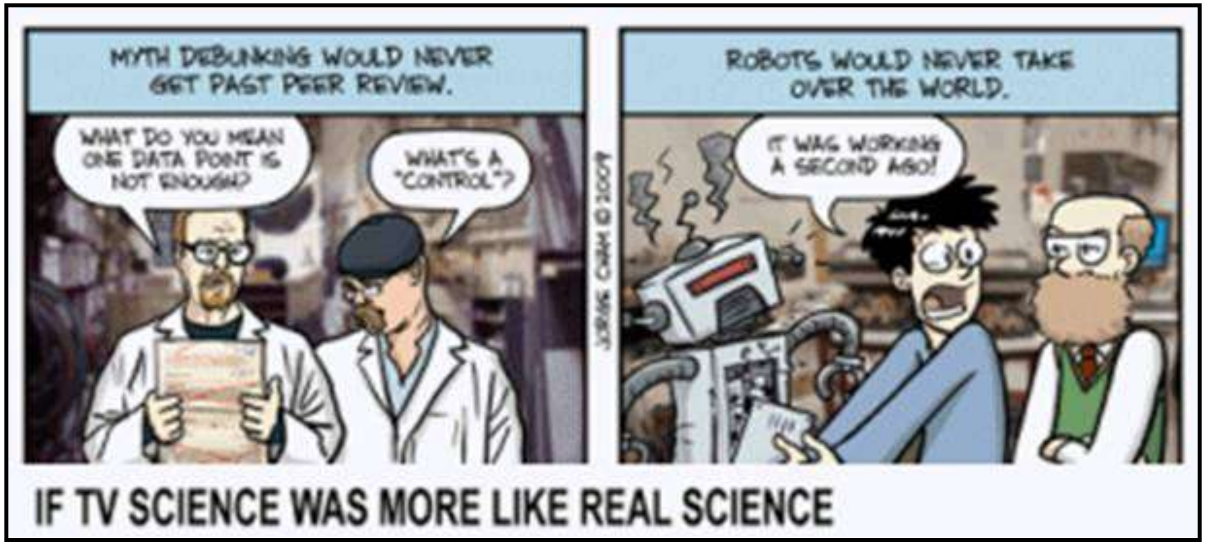

MYTH DEBUNKING WOULD NEVER GET PAST PEER REVIEW.
WHAT DO YOU MEAN ONE DATA POINT IS NOT ENOUGH?
WHAT'S A "CONTROL"?
JORGE CHAM © 2009
ROBOTS WOULD NEVER TAKE OVER THE WORLD.
IT WAS WORKING A SECOND AGO!
IF TV SCIENCE WAS MORE LIKE REAL SCIENCE

Feedback from last 2 lectures

- Bigger, better red boxes!
- Too much writing per slide
- More small boxes (equations etc) than large ones.

- Maybe too much text – more description, less reading out?

- More space in handouts for notes / revising

- Anything else?

Summary from last time:

- formal definitions of cross section
- definitions of Rutherford and Mott cross sections for coulomb scattering
- diffraction effects in scattering, determination of charge/mass distribution

For this lecture:

Implicit assumptions so far: nuclei are spheres

What determines the shape of a nucleus?

Answer: the attractive interactions between components form a droplet in order to minimize the number of “high energy” components at the surface (similar to the attractive forces between molecules in a droplet of water)

The binding energy of a nucleus:

- the energy available to hold nucleus together

Think of it this way:
- Take bunch of well-separated nucleons: binding energy is zero
- Bring them together: strong force glues them together. However, energy has to come from somewhere: binding energy must come from a **reduction** in nuclear mass

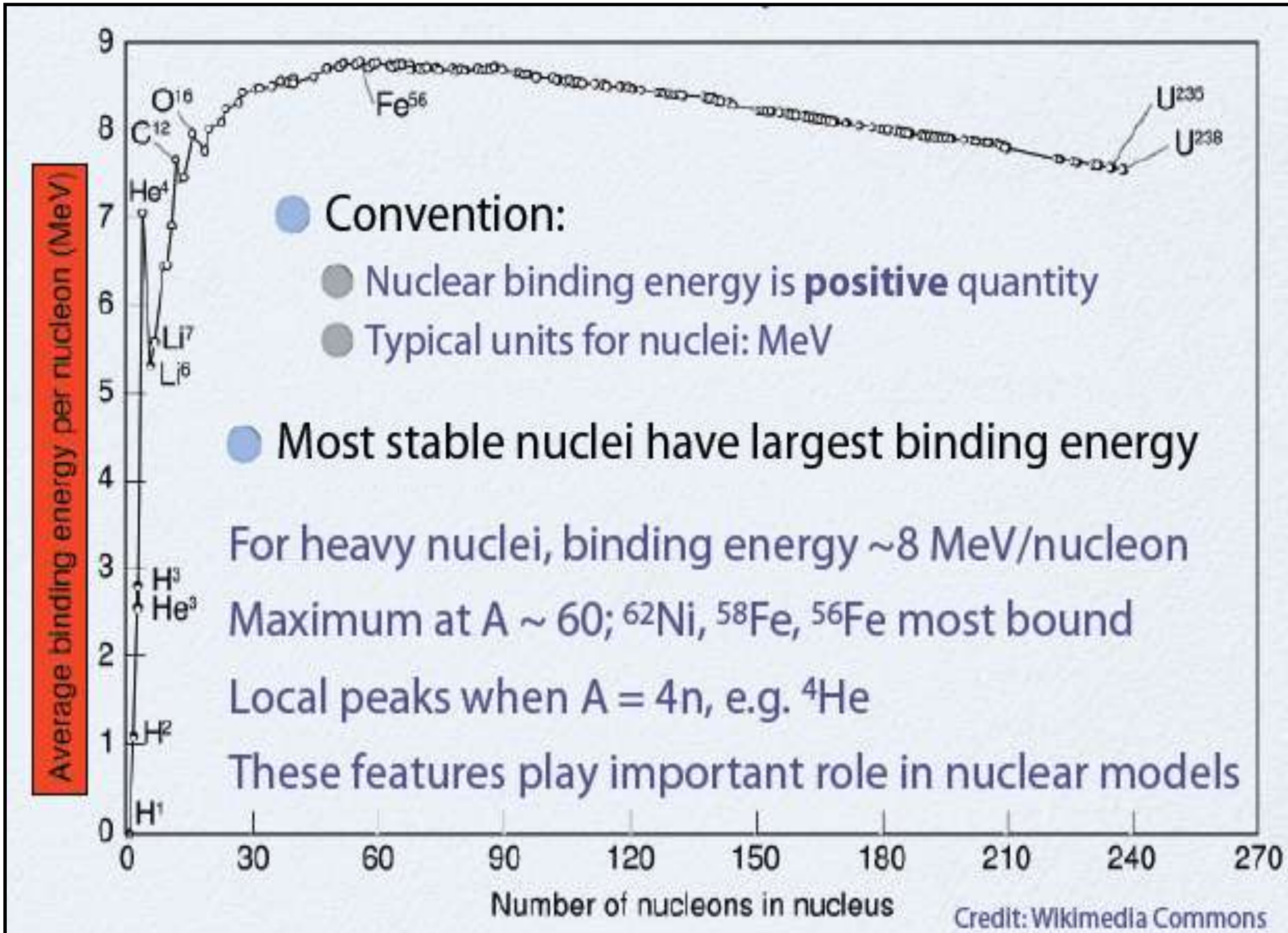
Formally, it is the difference between mass of component protons and neutrons and that of actual nucleus, related through $E = mc^2$:

$$B(A,Z) = Z\, m_p c^2 + N\, m_n c^2 - M(A,Z)\, c^2$$

Binding energy is a **positive quantity**
(don’t get confused here - the strong potential in which the nucleons sit is **negative**)

Binding energy per nucleon

→ the average energy state of nucleon is a sum of high energy “surface” nucleons with low energy “bulk” nucleons
→ nucleus minimizes energy by minimizing surface area – a sphere


Convention:
Nuclear binding energy is **positive** quantity
Typical units for nuclei: MeV
Most stable nuclei have largest binding energy
For heavy nuclei, binding energy ~8 MeV/nucleon
Maximum at A ~ 60; ^{62}Ni, ^{58}Fe, ^{56}Fe most bound
Local peaks when A = 4n, e.g. ^{4}He
These features play important role in nuclear models
Average binding energy per nucleon (MeV)
Number of nucleons in nucleus
H^1
H^2
H^3
He^3
He^4
Li^6
Li^7
C^{12}
O^{16}
Fe^{56}
U^{235}
U^{238}
0
1
2
3
4
5
6
7
8
9
0
30
60
90
120
150
180
210
240
270
Credit: Wikimedia Commons

<u>The liquid drop model</u>: a.k.a. **"semi empirical mass formula" [SEMF]**

Consider analogy with liquid drop:

- Liquids often considered as non-compressible
- Density constant, independent of radius

Then: radius $\mathbf{R \sim n^{1/3}}$ where n is number of molecules/nucleons in drop

Assume each molecule/nucleon bound with energy = -a
(i.e. energy required to remove from drop against force due to all others)

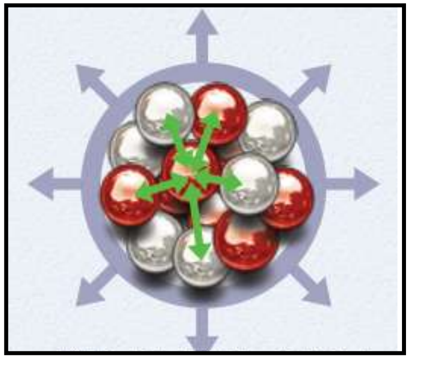
→ Total binding energy given by: $B = an$

However, only bound into drop on one side:
Will see reduction in potential energy ~ $\mathbf{4\pi R^2 T}$
i.e. proportional to surface area of drop and surface tension T

→ Using $R \sim n^{1/3}$ and substituting $\beta = 4\pi T$:

$$B = an - \beta n^{2/3}$$

"Bulk" and "surface" terms should have **<u>opposite</u>** sign: absence of strong force on other side of surface makes nucleus **<u>less</u>** stable, **<u>decreasing</u>** binding energy

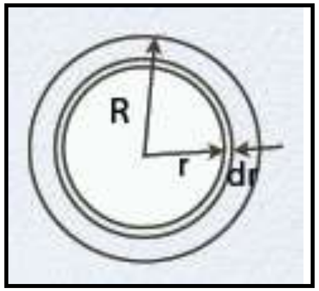


This describes basics of strong force interaction.
However, proton – proton repulsion also important.

→ Including electrostatics in the liquid drop model:

In charged drop, repulsive force acts to unbind drop - reduces binding energy

Get change in binding energy by calculating the electrostatic potential of charge Q, distributed uniformly throughout.



How to do it: take uniform charge density ρ. Start with sphere radius r and charge $4/3\pi\rho r^3$. Then consider thin shell from r to r+dr with charge $4\pi\rho r^2 dr$. Calculate work done to bring shell from infinity to radius r, then integrate this from r=0 to R and express it in terms of total charge.

The result:

$$U = \frac{3Q^2}{20\pi\epsilon_0 R}$$

→ Again using $R \sim n^{1/3}$ and substituting γ for all constants except charge:

$$B = an - \beta n^{2/3} - \gamma Q^2 n^{-1/3}$$

All nucleons (A) carry strong force; only protons (Z) charged
→ Substitute $n \rightarrow A$ and $Q \rightarrow Z$ in liquid drop model:

$$B(A,Z) = a_V A - a_S A^{2/3} - a_C Z^2 A^{-1/3}$$

Constants:

a_V for volume term

a_s for surface term

a_c for Coulomb term

Assumptions so far: 1) Nucleus is spherical
2) Nucleons behave like molecules in water drop:
→ Short-range attractive force holding them together with shorter-range repulsive force stopping collapsing onto each other
→ Nuclear density is constant

Just including bulk, surface and electrostatic terms: $B(A,Z) = a_v A - a_s A^{2/3} - a_c Z^2 A^{-1/3}$

However, formula inadequate as is:

→ For fixed A, binding energy maximised when Z = 0
Conversion of neutrons to protons allowed via β-decay, but $Z \rightarrow 0$ not seen in nature

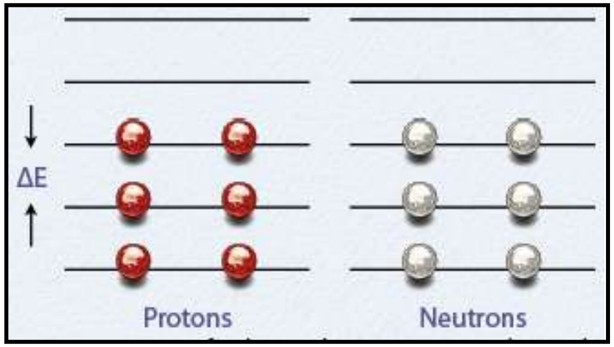
→ Why?? Model not complete: two **non-classical terms** required:

Both protons and neutrons are fermions - asymmetric wave functions and spin ½
Thus, they obey Pauli exclusion principle:

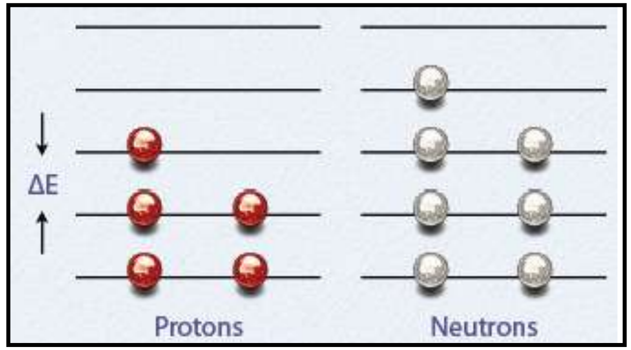
- Protons and neutrons must be arranged in separate energy levels: thus, consider two independent potential wells with identical energy levels, one for protons and one for neutrons

- Only two protons/neutrons possible in any given energy level, one with spin-up and the other with spin-down

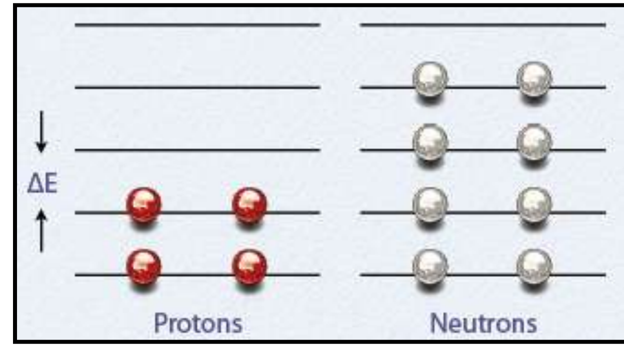
Breaking symmetry between Z and N takes energy:



Most stable configuration



N–Z = 2: Reduced p-p repulsion
But: moving proton has cost **ΔE**



N–Z = 4: has cost **2ΔE**

For more states:

N - Z	Step energy (ΔE)	Cumulative energy (ΔE)
2	1	1
4	1	2
6	3	5
8	3	8
10	5	13
12	5	18
14	7	25

Cumulative energy change from lowest energy N = Z given by $\sim (N - Z)^2 / 8 \times \Delta E$

This can be rewritten as $\sim \mathbf{(A - 2Z)^2 / 8 \times \Delta E}$

However, ΔE is not constant.
From QM: energy levels for particle in 3D finite well follow $\Delta E \sim 1 / R^3$. Again using $R \sim n^{1/3} = A^{1/3}$

→ $\mathbf{\Delta E \sim 1 / A}$

Collecting constants together, change in binding energy given by:

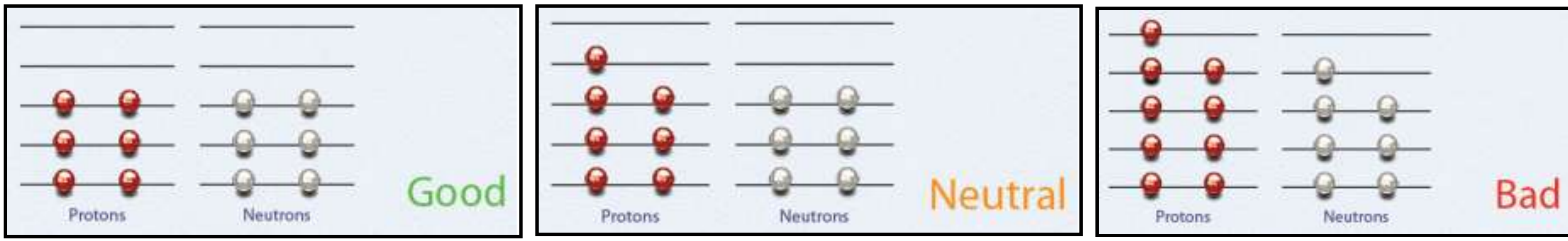
$$- a_a (A - 2Z)^2 A^{-1}$$

i.e. asymmetry between protons and neutrons again **reduces** the binding energy, hence -ve

Spin pairing in the liquid drop model:

Spin pairing favours pairs of fermionic nucleons (similar to electrons in atoms)
i.e. a pair with opposite spin have lower energy than pair with same spin

Best case: even numbers of both protons and neutrons
Worst case: odd numbers of both protons and neutrons
Intermediate cases: odd number of protons, even number of neutrons or vice versa



→ Subtract small energy δ required to decouple nucleons from binding energy:

$$\begin{aligned} \delta &= +a_p A^{-1/2} && \text{for both N \& Z odd} \\ &= 0 && \text{for N even, Z odd / Z even, N odd} \\ &= -a_p A^{-1/2} && \text{for both N \& Z even} \end{aligned}$$

→ a_p collects constants, $A^{-1/2}$ dependence provides best empirical fit to data

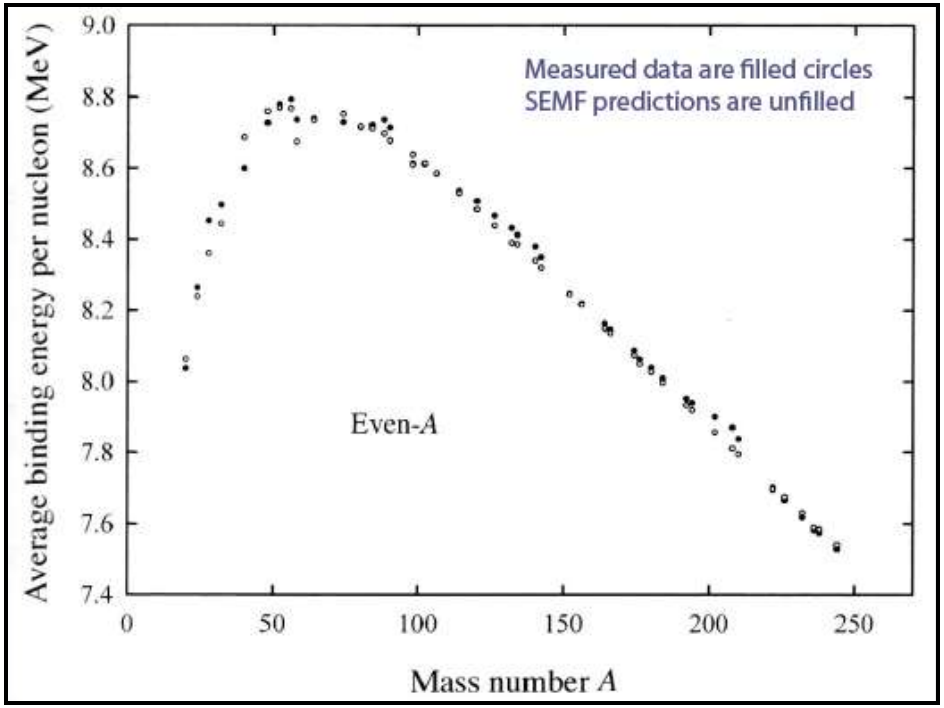
→ subtracting δ **reduces** BE for N and Z both odd

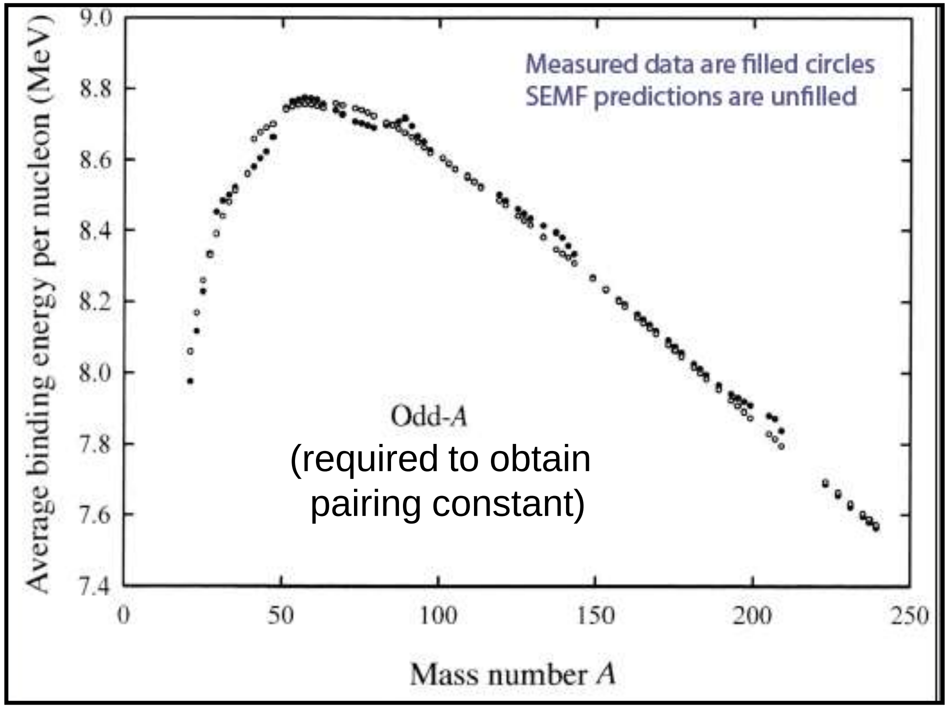
→ subtracting δ **adds** small amount to BE for N and Z both even

The constants:

Why semi-empirical?
Constants not defined on basis of theory

Derived by fitting to measured masses
For A > 20, accuracy generally better than 0.1 MeV (i.e. error < 1% of total mass)





- a_v = 15.56 MeV
- a_s = 17.23 MeV
- a_c = 0.697 MeV
- a_a = 23.285 MeV
- a_p = 12.0 MeV

The largest contributors to BE (first three)
(multiply by A, $A^{2/3}$ and $Z^2 / A^{1/3}$ respectively)

Smallish contribution to BE (last two)
(no real multiplicative effect)

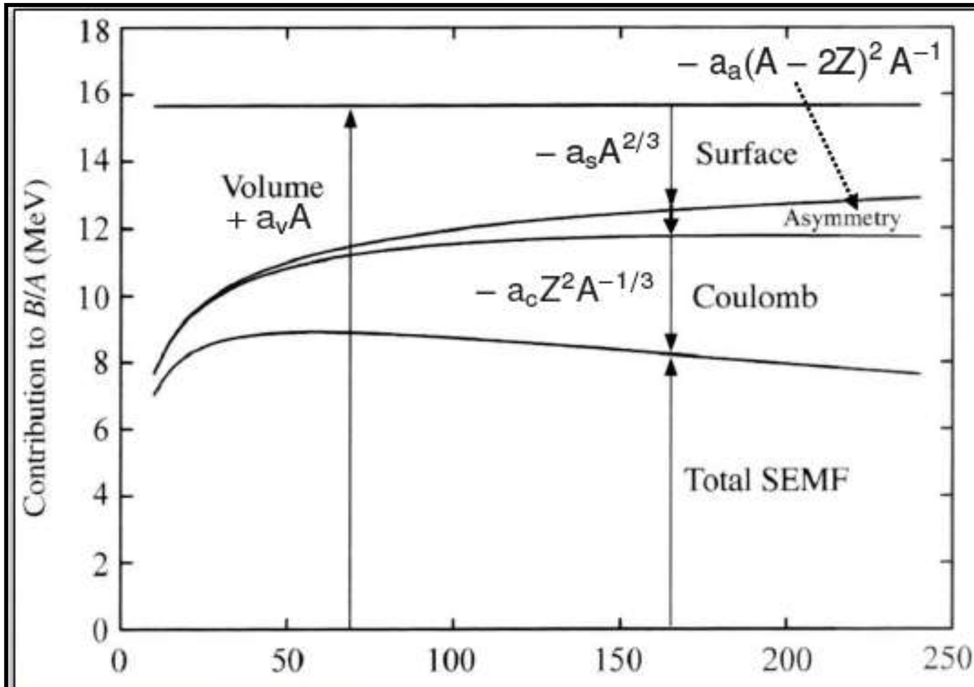
(Note: different authors list slightly different constants)

Summary of terms in the liquid drop model:

$$\delta = +a_pA^{-1/2} \quad \text{for N \& Z odd}$$
$$= 0 \quad \text{for N even, Z odd / Z even, N odd}$$
$$= -a_pA^{-1/2} \quad \text{for N \& Z even}$$

$$B(A,Z) = \underbrace{a_vA}_{\text{Volume term}} - \underbrace{a_sA^{2/3}}_{\text{Surface term}} - \underbrace{a_cZ^2A^{-1/3}}_{\text{Coulomb term}} - \underbrace{a_a(A-2Z)^2A^{-1}}_{\text{Asymmetry term}} - \underbrace{\delta}_{\text{Pairing term}}$$

- binding energy "comes" from volume term
(i.e. the strong interaction between nucleons)
- all other terms reduce binding energy:



Pairing term neglected here

Can also write in terms of mass:

$$M(A,Z)\,c^2 = Z\,m_pc^2 + N\,m_nc^2 - B(A,Z)$$

(semi empirical **mass** formula)

■ Good prediction of nuclear masses: assumptions must be reasonable

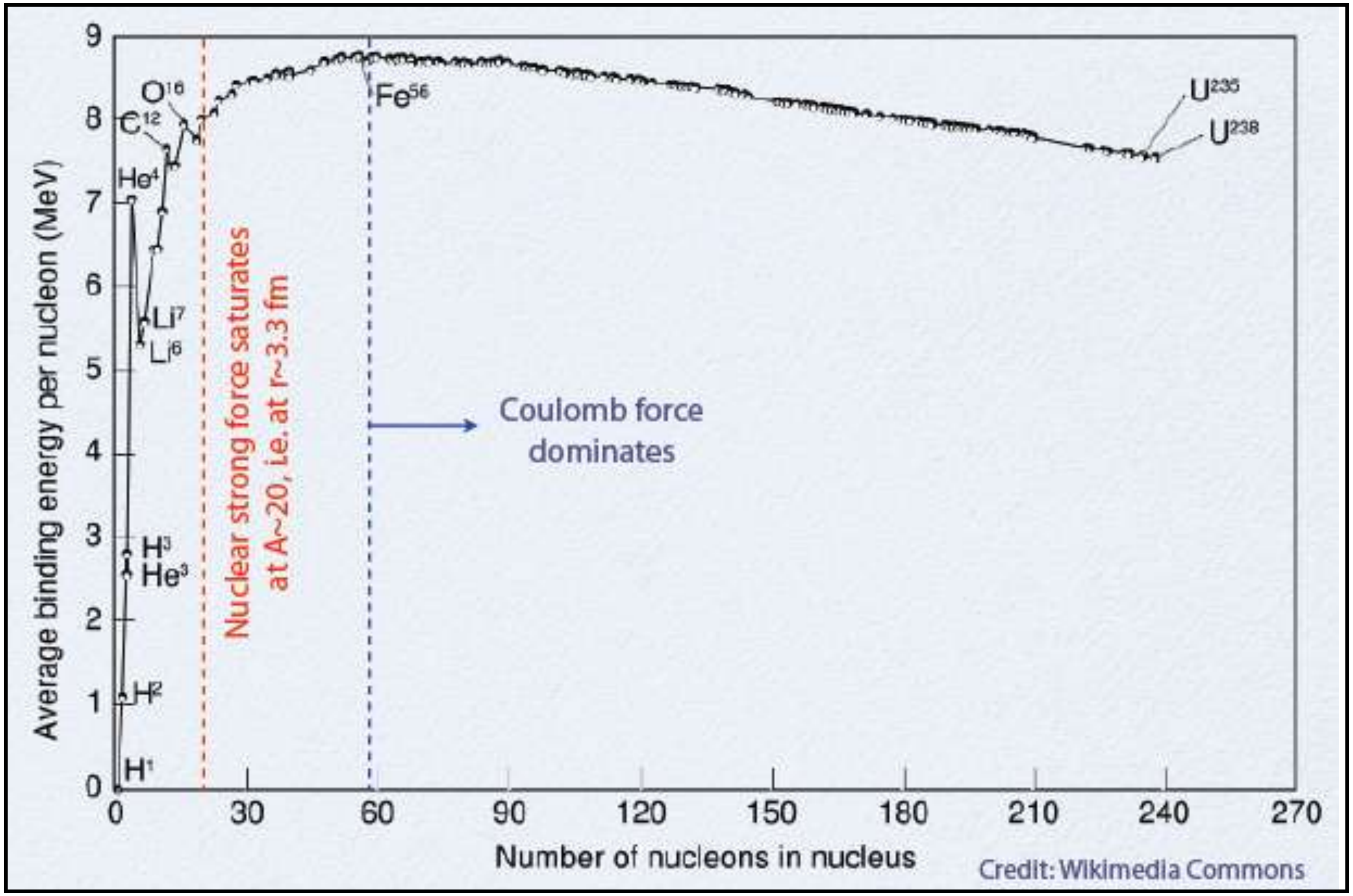
Implications of the liquid drop model for the nuclear force:

The attractive (volume) term is proportional to A, so strong force must be short range. If every nucleon interacted with every other nucleon, would go as $\sim A^2$ for large A.

→ inter-nucleon force saturates very quickly on increasing A

→ must also become repulsive at small separations (prevents collapse of nucleus)

By contrast, electrostatic component goes as $\sim Z^2$ for large Z, i.e. does **not** saturate:



Typical exam questions:

1) Calculate the binding energy of uranium $^{238}_{92}\mathrm{U}$

A = 238
Z = 92
N = 238 - 92 = 146

Binding energy terms (in MeV):
Volume: $a_v A = 15.56 \times 238 = 3703.28$
Surface: $a_s A^{2/3} = 17.23 \times (238)^{2/3} = 661.71$
Coulomb: $a_c Z^2 A^{-1/3} = 0.697 \times (92)^2 \times (238)^{-1/3} = 951.95$
Asymmetry: $a_a (A-2Z)^2 A^{-1} = 23.285 \times (238 - 184)^2 \times 238^{-1} = 285.29$
Pairing: $a_p A^{-1/2} = 12.0 \times (238)^{-1/2} = 0.78$

B = +Volume - Surface - Coulomb - Asymmetry + Pairing (N, Z even)
= 3703.28 - 661.71 - 951.95 - 285.29 + 0.78
= **1805.10 MeV**

→ correct to 0.19%

2) What percentage is the binding energy of the total nuclear mass?

Masses of unbound nucleons: p = 938.27 MeV/c^2, n = 939.57 MeV/c^2
Protons: 92 x 938.27 = 86 320.84 MeV/c2
Neutrons: 146 x 939.57 = 137 177.22 MeV/c2
Total: = 223 498.06 MeV/c2

Mass of bound nucleus:
= Mass of unbound nucleons - Binding energy = 223 498.06 - 1805.10 = **221 692.96 MeV/c^2**

→ BE is < 1% of total mass energy of nucleus

Another example – Energy given out by nuclear reaction:

Calculate the energy released when an alpha particle is emitted by ^{235}U nucleus.

The reaction: $^{235}U \rightarrow \alpha + {}^{231}Th$

Need to know binding energies of all components in reaction:

^{235}U : B(235,92) = 3642.5 - 639.766 - 976.8 - 254.566 = 1771.368 MeV

^{231}Th : B(231,90) = 3580.5 - 632.486 - 939.928 - 258.974 = 1749.113 MeV

However, mass of alpha particle not well predicted by SEMF
- it is a particularly stable "magic" nucleus (see later)
- in practice, simply need to be given it!

$\rightarrow BE_{\alpha} = 28.3\ MeV/c^2$

Energy released $= BE_{\alpha} + B(A-4,Z-2) - B(A,Z)$

$= 28.3 + 1749.1 - 1771.4 = 6.0$ MeV

→ This energy is released as kinetic energy of α particle

Summary:

- explain and understand each of the terms in the liquid drop model
- be able to calculate nuclear masses and binding energies

For next time:

Limitations of the liquid drop model