# STATISTICS-I

# NORMAL DISTRIBUTION:

**A random variable of the continuous type that has a pdf of the form of**

$$f(\mathbf{x})=\frac{1}{b\sqrt{(2\pi)}}\ \exp\left[\frac{-(x-a)^2}{2b^2}\right]\ -\infty < x < \infty$$

**is said to have a normal distribution and any f(x) of this form is called a normal pdf.It is denoted by N(a,b²).**

**The m.g.f of a normal distribution is as follows**

$$\mathbf{M(t)=E(e^{tx})}$$

$$=\int_{-\infty}^{\infty} e^{tx} f(x)dx$$

$$=\int_{-\infty}^{\infty} e^{tx} \frac{1}{b\sqrt{(2\pi)}} \exp\left[\frac{-(x-a)^2}{2b^2}\right]dx$$

$$=\int_{-\infty}^{\infty} \frac{1}{b\sqrt{(2\pi)}} \exp\left[-\left(\frac{-tx+(x-a)^2}{2b^2}\right)\right]dx$$

$$= \int_{-\infty}^{\infty} \frac{1}{b\sqrt{(2\pi)}} \exp[-( \frac{-tx+x^2-2ax+a^2}{2b^2} )]dx$$

$$= \int_{-\infty}^{\infty} \frac{1}{b\sqrt{(2\pi)}} \exp[-( \frac{-2b^2tx+x^2-2ax+a^2-(a-b^2t)^2+(a+b^2t)}{2b^2} )]dx$$

SINCE $\frac{-2b^2tx+x^2-2ax+a^2-(a+b^2t)^2+(a+b^2t)^2}{2b^2}$

$$= \frac{-2b^2tx+x^2-2ax+a^2-(a^2+b^4t^2+2ab^2t)+(a^2+b^4t^2+2ab^2t)}{2b^2}$$

$$= \frac{-2b^2tx+x^2-2ax+a^2-a^2-(b^2t)^2-2ab^2t+a^2+(b^2t)^2+2ab^2t}{2b^2}$$

$$= \frac{x^2+a^2+(b^2t)^2-2xa+2ab^2t-2b^2tx-a^2+a^2-b^2(t)^2-2ab^2t}{2b^2}$$

$$= \frac{(x-a-b^2t)^2-(b^2t)^2-2ab^2t}{2b^2}$$

$$\therefore \int_{-\infty}^{\infty} \frac{1}{b\sqrt{(2\pi)}} \exp\left[\frac{-(x-a-b^2t)-[(b^2t)^2+2ab^2t]}{2b^2}\right]dx$$

$$=\exp\left[\frac{(b^2t)^2+2ab^2t}{2b^2}\right] \int_{-\infty}^{\infty} \frac{1}{b\sqrt{(2\pi)}} \exp\left[\frac{(-(x-a-b^2t)}{2b^2}\right]dx$$

$$=\exp\left[\frac{(b^2t^2)+2ab^2t}{2b^2}\right](1)$$

Since $\int_{-\infty}^{\infty} \frac{1}{b\sqrt{(2\pi)}}$

**Because a replaced by a+b²t**

$$=\exp\left[\frac{(b^2t)^2}{2b^2}+\frac{2ab^2t}{2b^2}\right]$$

$$=\exp\left[\frac{b^2t^2}{2}+at\right] \Rightarrow \exp\left[at+\frac{b^2t^2}{2}\right]$$

Now differentiation M(t) with respect to t

$$M(t)=e^{(at\frac{b^2t^2}{2})}$$

$M'(t)=e^{(at+\frac{b^2t^2}{2})}(a+b^2t)$

$M''(t)=e^{(at+\frac{b^2t^2}{2})}(b^2)+e^{(at+\frac{b^2t^2}{2})}(a+b^2t)^2$

Mean $\mu=M'(0)$

$M'(0)=e^{(0)}(a)$

$=a$

Variance $\sigma^2=M''(0)-(M'(0))^2$

$\sigma^2=b^2+a^2-a^2=b^2$

Thus take normal p.d.f is written in the form

of $f(x)=\frac{1}{\sigma\sqrt{(2\pi)}}\exp[\frac{-(x-\mu)^2}{2\sigma^2}]\ -\infty < x < \infty$ and m.g.f can be written in the form $M(t)=\exp[\mu t+\frac{\sigma^2t^2}{2}]$

∴ It is denoted by $N(\mu,\sigma^2)$.

**EXAMPLE:**

**If X has the m.g.f** $M(t)= e^{2t+32t^2}$ **Then X has a normal distribution with** $\mu=2, \sigma^2=64$ **then find the p.d.f Of the distribution.**

**Solution:**

**Here** $\mu=2, \sigma^2=64$ **and X has the normal distribution**.

$$\therefore f(x)= \frac{1}{\sigma\sqrt{(2\pi)}} \exp[\frac{-(x-\mu)^2}{2\sigma^2}]$$

$$= \frac{1}{8\sqrt{(2\pi)}} \exp[\frac{-(x-2)^2}{2\times 64}]$$

$$= \frac{1}{8\sqrt{(2\pi)}} \exp[\frac{-(x^2+4-4x)}{128}]$$

**X~N(0,1) then f(x),**

$$F(x)= \frac{1}{\sqrt{(2\pi)}} \exp[\frac{-(X-0)^2}{2}]$$

$$= \frac{1}{\sqrt{(2\pi)}} \exp[\frac{-X^2}{2}]$$

$$M(t) = \exp[\mu t + \frac{\sigma^2 t^2}{2}]$$

$$= \exp[\frac{t^2}{2}].$$

**THEOREM:**

**If the random variable X is** $N(\mu,\sigma^2), \sigma^2 > 0$

**Then the random variable** $W = \frac{(X-\mu)}{\sigma}$ **is** N(0,1).

**Proof:**

**The distribution function** G(W) **of** W **is**

$G(W) = P_r(W \leq w)$

$= p_r\left(\frac{(X-\mu)}{\sigma} \leq w\right)$

$= p_r(X \leq w\sigma + \mu)$

$\therefore G(w) = \int_{-\infty}^{w\sigma+\mu} f(x)\,dx$

Since $f(x) = \frac{1}{\sigma\sqrt{(2\pi)}} \exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right], \sigma > 0$

$= \int_{-\infty}^{w\sigma+\mu} \frac{1}{\sigma\sqrt{(2\pi)}} \exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right] dx$

If we change the variable of integration by $y = \frac{(x-\mu)}{\sigma}$

$\Rightarrow x = y\sigma + \mu \quad dx = dy\sigma$

$X = w\sigma + \mu$

$$Y = \frac{(w\sigma + \mu - \mu)}{\sigma}$$

$$\therefore G(w) = \int_{-\infty}^{w} \frac{1}{\sigma\sqrt{(2\pi)}} \exp\left[\frac{-(Y)^2}{2}\right] \sigma dy$$

$$= \int_{-\infty}^{w} \frac{1}{\sqrt{(2\pi)}} e^{\frac{-y^2}{2}} dy$$

$\therefore$ The pdf $g(w) = G'(w)$ of the continuous type random variable is

$$g(w) = \frac{1}{\sqrt{(2\pi)}} e^{\frac{-w^2}{2}} \quad -\infty < w < \infty$$

Thus w is N(0,1).

# THANK YOU