Poisson distribution

Definition

A random variable X with pdf given

$f(x)=(m^{x} e^{-m})/x\,!, x=0,1,\ldots.$

$\quad =0 \qquad , \text{elsewhere}$

where m>0 is said to have a poission distribution & f(x)is called "poission distribution". The constant m is called the parameter of the distribution.

REMARK :

the series $1+m/1! + m^2/2! + \ldots = \sum_{x=0}^{\infty} m^x/x!$ converges for are values of m to e

$m>0$, $f(x)>=0$ &

$$\sum_{x=0}^{\infty} f(x) = \sum_{x=0}^{\infty} (m^x e^{-m})/x! = e^{-m} \sum_{x=0}^{\infty} (m^x)/x!$$

$$= e^{-m} (1+m/1! + m^2/2! + \ldots)$$

$$= e^{-m} e^{m} = 1$$

i.e.)f(x) satisfies condition of being pdf of a discrete type of random variable.

The mgf of the poission distribution is given by $m(t)=E(e^{tx})$

$$= \Sigma e^{tx} f(x)$$

$$= \Sigma e^{tx} (me^{-m})/x!$$

$$= e^{-m} \Sigma ((me^{t)x})/x!$$

$$= e^{-m} (1+m/1! + m^2/2! + \ldots)$$

$$= e^{-m} e^{met}$$

$$= e^{-m} (e^{mt-1})$$

for all real values of t

$m'(t) = e^{m}(e^{mt-1}) me^{t}$

$m''(t) = e^{m}(e^{mt-1}) me^{t} + e^{m}(e^{mt-1})(me^{t})^{2}$

mean=m'(t)=m & =m''(t)- ^{2} =m+m^2 -m^2 =m

i.e.) a poission distribution has ==m>0

the poission pdf is written as

$f(x) = \mu^{x} e^{-\mu} / x!$,x=0,1.2.....

=0 , elsewhere.

Eg 1:
suppose that x has poission distribution with $\mu=2$. Then find pdf and compute $pr[1\le x]$.

Eg 2:
If the mgf of a random variable is $m(t)=e^{4(e^t-1)}$. Then x has a poission distribution with μ=4.

eg 3:
let the probability of exactly one blemish in 1 feet of wire be about 1/1000 & let the probability of 2 or more blemishes in that length be for all practical purpose zero.

The end