



# Poisson distribution



# *Definition*



A random variable  $X$  with pdf given

$$f(x) = \frac{m^x e^{-m}}{x!}, x=0, 1, \dots$$

$= 0$  , elsewhere

where  $m > 0$  is said to have a poisson distribution &  $f(x)$  is called “poission distribution”. The constant  $m$  is called the parameter of the distribution.

## REMARK :

the series  $1 + m/1! + m^2/2! + \dots = \sum_{x=0}^{\infty} m^x/x!$  converges for all values of  $m$  to  $e^m$   
 $m > 0, f(x) \geq 0$  &

$$\begin{aligned}\sum_{x=0}^{\infty} f(x) &= \sum_{x=0}^{\infty} (m^x e^{-m})/x! = e^{-m} \sum_{x=0}^{\infty} (m^x)/x! \\ &= e^{-m} (1 + m/1! + m^2/2! + \dots) \\ &= e^{-m} e^m = 1\end{aligned}$$

i.e.)  $f(x)$  satisfies condition of being pdf of a discrete type of random variable.

The mgf of the poisson distribution is given by  $M(t) = E(e^{tx})$

$$\begin{aligned} &= \sum e^{tx} f(x) \\ &= \sum e^{tx} (m^x e^{-m})/x! \\ &= e^{-m} \sum ((me^t)^x)/x! \\ &= e^{-m} (1 + me^t/1! + m^2 e^{2t}/2! + \dots) \\ &= e^{-m} e^{met} \\ &= e^{-m} (e^{mt-1})\end{aligned}$$

for all real values of  $t$

$$m'(t) = e^m (e^{mt-1}) me^t$$

$$m''(t) = e^m (e^{mt-1}) me^t + e^m (e^{mt-1}) (me^t)^2$$

$$\text{mean} = m'(t) = m \quad \& \quad \text{variance} = m''(t) - (m')^2 = m + m^2 - m^2 = m$$

i.e.) a poisson distribution has  $\mu = m > 0$

the poisson pdf is written as


$$f(x) = \frac{\mu^x e^{-\mu}}{x!} \quad , x=0, 1, 2, \dots$$

= 0, elsewhere.



**Eg 1:**

**suppose that  $x$  has poisson  
distribution with  $\mu=2$ . Then  
find pdf and compute  
 $\text{pr}[1 \leq x]$ .**



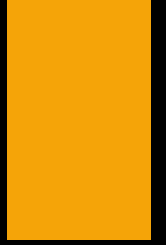
Eg 2:

If the mgf of a random variable is  $m(t) = e^{4(e^t - 1)}$ . Then  $x$  has a poisson distribution with  $\mu = 4$ .

eg 3:

let the probability of exactly one blemish in 1 feet of wire be about  $1/1000$  & let the probability of 2 or more blemishes in that length be for all practical purpose zero.

# The end



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