I M.Sc., Physics	Major Paper- 2	Marks :100
Semester I	**Classical Mechanics**	Hrs/Week :6
Code:		INT:25, EXT:75

- **To know classical mechanical methods and theories**
- **To understand classical mechanical transformations, oscillations and concepts**
- **To apply them to solve physics problems**

***Unit –I* Lagrangian and Hamiltonian methods**

Generalized coordinates - Lagrangian equation of motion- Variational principle and Lagrangian equation of motion – Hamiltonian equation of motion – Cyclic coordinates and Routh's procedure – Physical significance of the Hamiltonian – Hamiltonian equations form variational principle-The principle of least action - Simple applications.

***UNIT –II* Central field motion**

Motion under a central force – General features of central force motion- Reduction of two body central force problem to the equivalent one body problem- Equation of amotion in a central field. Equation of orbit in a central field- condition for closed orbit (Bertrand's theorem)- The virial theorem- Kepler's law of planetary motion-scattering in a central force field- Rutherford's Alpha Particle Scattering.

***Unit III* Canonical Transformations**

The equation of Canonical Transformations - examples of Canonical Transformations – Harmonic Oscillator- Lagrange and Poisson bracket – Equation of motion in Poisson bracket notation- Liouville's theorem.

Unit-IV Small oscillations

Formulation of the Problem-Eigen value equation and the principle axes Transformation-Frequencies of free vibrations and normal Coordinates-Free vibrations of a linear triatomic molecule and some macroscopic applications.

Unit –V Hamilton- Jacobi theory

Hamilton-Jacobi equation – Applications: Harmonic Oscillator and Kepler's Problem – The Hamilton –Jacobi equation for Hamilton's characteristic's function-Action and Angle variables- Harmonic Oscillator problem using action and angle variables- Kepler's problem in action- Angle variable

TEXT BOOK:

1. Classical Mechanics, H. Goldstein, II edn. (1980, Narosa). World student Edn

   Chapter: 3, 6 ,8,9,10 revelent sections.

REFERENCE BOOKS:

1. Mechanics, L.D. Laudau and E.M. Lifshitz
2. Classical Mechanics, T.W.B. Kibble
3. Classical Mechanics, N.C. Rana and P.S. Juog

I M.Sc., Physics	Major Paper- 3	Marks :100
Semester I	APPLIED ELECTRONICS	Hrs/Week :6
Code:		INT:25, EXT:75

COURSE OBJECTIVES:

- To be familiar with various semiconductor devices and amplifier systems then it will be give us a resonable mathematical simplification of the problem.

Generalised co-ordinates are designated by the latter q with numerical Subscribes $q_1, q_2, q_3 \dots q_n$ represence a set of n generalised co-ordinates otherwise we can be written as,

$$q_j \; (j = 1, 2 \dots n)$$

Thus when a particle moves in a plane it can be either describe in cartesian co-ordinates (x, y) or polar co-ordinates (r, θ)

$$q_1 = r = \sqrt{x^2+y^2}$$

$$q_2 = \theta = \tan^{-1}(y/x)$$

Spherical polar co-ordinates

$$q_1 = r = \sqrt{x^2+y^2+z^2}$$

$$q_2 = \theta = \cot^{-1}\left(\frac{z}{(x^2+y^2)^{1/2}}\right)$$

$$q_3 = \phi = \tan^{-1}(y/x)$$

In general we can always express the generalised co-ordinates as some function of Cartesian co-ordinates and also function of time.

$$\left.\begin{aligned} q_1 &= q_1(x_1, y_1, z_1; x_2, y_2, z_2; \ldots t) \\ q_2 &= q_2(x_1, y_1, z_1; x_2, y_2, z_2; \ldots t) \\ &\ldots\ldots \\ q_n &= q_n(x_n, y_1, z_n; x_{2n}, y_{2n}, z_{2n}; \ldots t) \end{aligned}\right\} \rightarrow n$$

For a system of N particle free from constrains we require 3N generalised co-ordinates.

Therefore the no. of transformation equation set of 3N cartesian co-ordinates to 3N generalised co-ordinates are shown in above the equ (n)

It is also possible to express cartesian co-ordinates in term of generalised co-ordinates.

$$x_1 = x_1 (q_1, q_2 \ldots q_{3N}, t)$$
$$y_1 = y_1 (q_1, q_2 \ldots q_{3N}, t)$$
$$\ldots$$
$$z_n = z_n (q_1, q_2 \ldots q_{3N}, t)$$

System	Generalised co-ordinates
1. Simple pendulum	θ = The angle which the pendulum makes with vertical line ~~Hand~~ point of Suspension.
2 Fly wheel	θ = The angle between a definite radius of the Flywhe and Fixed line perpendicula to the axis.
3 Particle on the Surface of the sphere	θ, ϕ - The usual polar angle of the sphere
4. beads of an abacus	x - The Cartesion co-ord along the horizontal vial
5 Hydrogen molecule	x, y, z, ϕ, ψ - where x, y, z are Cartesion coordi of the centre of molecule ϕ are the angle of rotation abo to mutually perpendicular axi through Centre

6. particles moving on inside surface of the cone

r, θ - where r is the radius vector drawn from the origin to the point of the particle and the angle θ of the radius vector with the fixed slant edge of the cone.

Force - ML

Generalised displacement

Let us consider a small displacement of a particle of a system defined by change δr_i ($i = 1, 2 \ldots N$) with the time t held fixed

$r_i = r_i(q_1, q_2, \ldots, q_{3N}, t)$

by Euler's theorem

$$\delta r_i = \sum_{j=1}^{3N} \frac{\partial r_i}{\partial q_j} \delta q_j \quad [\because \delta t = 0] \; t \text{ is constant}$$

δq_j's are called generalised displacement or otherwise virtual orbitrary displacement. If q_j is a angle co-ordinate then δq_j is a angular displacement.

Generalised velocity

Velocity may be describe interm of time derivative $\dot{q}_j$ of the generalised Co-ordinate q_j which is then called generalised velocity associated with the particular Co-ordinate q_j. Therefore for a unconstrained system,

$$r_i = r_i(q_1, q_2, \ldots q_{3N}, t)$$

$$\dot{r}_i = \sum_{j=1}^{3N} \frac{\partial r_i}{\partial q_j} \cdot \dot{q}_j + \frac{\partial r_i}{\partial t}\left(\frac{dt}{dt}\right)$$

$$\dot{r}_i = \sum_{j=1}^{3N} \frac{\partial r_i}{\partial q_j} \cdot \dot{q}_j + \frac{\partial r_i}{\partial t}$$

If the N system contains k Constrains then the no. of generalised Co-ordinates is $3N - k = f$

$$\dot{r}_i = \sum_{j=1}^{f} \frac{\partial r_i}{\partial q_j} \cdot \dot{q}_j + \frac{\partial r_i}{\partial t}$$

∴ If the generalised co-ordinate q_j has the dimentions of momentum the generalised velocity will have dimens of the force. If the generalised co-ordinate of the dimension of angular co-ordinate has corresponding generalised velocity angular velocity

[Generalised angular momentum L
The generalised velocity $\frac{dL}{dt}$ = torque]

Generalised momentum

The momentum associated with generalised co-ordinate q_j is defined as generalised momentum p_j associated with the co-ordinate q_j.

$$p_j = \frac{\partial T}{\partial \dot{q}_j}$$ (T is K.E of the system)

p_j need not always have the dimension of linear momentum [MLT^{-1}]. If q_j happens to be angular co-ordinate Then p_j is corresponding angular momentum with dimension ML^2T^{-1}

Generalised Force

The definition of generalised force associated with the generalised displacement is given as follows.

Let us consider the amount of work done by a force $\sum_i F_i$ on the system during an arbitrary small displacement $\sum_i \delta r_i$ of the system.

$$\delta W = \sum_{i=1}^{N} F_i \cdot \delta r_i$$

$$= \sum_{i=1}^{N} F_i \cdot \sum_{j=1}^{3N} \frac{\partial r_i}{\partial q_j} \delta q_j$$

$$= \sum_{j=1}^{N} Q_j \cdot \delta q_j$$

$$Q_j = \sum_{i=1}^{N} F_i \frac{\partial r_i}{\partial q_j}$$

Generalised Force

Q_j depends on force acting on the particle and on the co-ordinate q_j and also time t. whatever dimension a generalised co-ordinate has the product of generalised force and generalised displacement must have the dimensions of work.

which implies the generalised force need not have the dimensions of force.

Hamilton's Variational principle

The mechanical problem involving two particles every particle by a set of three co-ordinates can be reduce to a single particle problem by considering as a single particle moving in a six dimensional space.

~~Thus~~ in general a problem involving n particles can be treated as one of the single particle moving along a tragectry in ~~the~~ 3N dimensional space.

This space is refered to configuration space and the single partic as system point. motion of system point in configuration space is called the mo of system between any two given instan hence configuration space have no necessary connection with the real three dimensional space.

∴ The principle states that

$\int_{t_1}^{t_2} (T-V)\,dt$ shall have stationary value [extremum] where T is K.E of the system and is a function of co-ordinates and their derivatives and V the P.E of the system is a function of co-ordinates only.

Such a system for which V is purely a function of co-ordinates is called conservative system.

<u>The statement of hamilton principle</u>

Hamilton principle for a conservative system is stated as follows. The motion of system from time t_1 to t_2 is such that the line integral $I = \int_{t_1}^{t_2} L\,dt$ where $L = T - V$ is extremum for the path of the motion

Deduction of Lagrangies equation of motion from variational principle

Let us consider a conservative system of particles employing the generalised co-ordinates the integral can be written as

$$\int_{t_1}^{t_2} [T(q_j, \dot{q}_j) - V(q_j)]\, dt$$

According to hamilton's variation principle

$$\delta \int_{t_1}^{t_2} [T(q_j, \dot{q}_j) - V(q_j)]\, dt = 0$$

$$\int_{t_1}^{t_2} \left[\left(\sum_j \frac{\partial T}{\partial q_j} \delta q_j + \frac{\partial T}{\partial \dot{q}_j} \delta \dot{q}_j\right) - \sum_j \frac{\partial V}{\partial q_j} \delta q_j\right] dt = 0$$

$$\int_{t_1} \left[\sum_j \left[\frac{\partial T}{\partial q_j} - \frac{\partial V}{\partial q_j}\right] \delta q_j + \sum_j \frac{\partial T}{\partial \dot{q}_j} \delta \dot{q}_j\right] dt = 0$$

$$\delta \dot{q}_j = \frac{d}{dt} [\delta q_j$$

$$\int_{t_1}^{t_2} \sum_j \left(\frac{\partial T}{\partial q_j} - \frac{\partial V}{\partial q_j}\right) \delta q_j \, dt + \int_{t_1}^{t_2} \sum_j \frac{\partial T}{\partial \dot{q}_j} \frac{d}{dt} (\delta q_j) \, dt = 0$$

Integrating the parts of second terms

$\int u v = uv - \int v du$

$$\int_{t_1}^{t_2} \sum_j \left(\frac{\partial T}{\partial q_j} - \frac{\partial V}{\partial q_j}\right) \delta q_j \, dt + \sum_j \frac{\partial T}{\partial \dot{q}_j} \delta q_j \Big|_{t_1}^{t_2} - \int_{t_1}^{t_2} \delta q_j \sum_j \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) dt =$$

Since in a such a variation there is no co-ordinate variation at end points

$$\delta q_j \Big|_{t_1}^{t_2} = 0$$

The equation reduces to,

$$\int_{t_1}^{t_2} \sum_j \left(\frac{\partial T}{\partial q_j} - \frac{\partial V}{\partial q_j}\right) \delta q_j \, dt - \int_{t_1}^{t_2} \sum_j \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) \delta q_j \, dt = 0$$

$$\int_{t_1}^{t_2} \sum_j \left[\frac{\partial T}{\partial q_j} - \frac{\partial V}{\partial q_j} - \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)\right] \delta q_j \, dt = 0$$

Since each δq_j is are independent of eachother the co-efficient of every δ

should be equated to zero to satisfy above equation. Thus equating the co-efficients of j^{th} co-ordinate to zero we get,

$$\frac{\partial T}{\partial q_j} - \frac{\partial V}{\partial q_j} - \frac{d}{dt}\left(\frac{\partial V}{\partial \dot{q}_j}\right) = 0$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial}{\partial q_j}(T-V) = 0$$

$$\frac{d}{dt}\left(\frac{\partial (T-V)}{\partial \dot{q}_j}\right) - \frac{\partial}{\partial q_j}(T-V) = 0$$

$$\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0}$$

$\because L = T-V$

[For a conservative system V is not func of $\dot{q}_j$ only on q_j]

This is the lagrangian equation of motion for a conservative system.

Note: Configuration space is a venble device introduce for displaying the motion of a sy lagrangian approach]

For a non conservative system the potential are velocity dependent and called generalised potential represented by the letter $U(q_j,$

$$Q_j = -\frac{\partial U}{\partial q_j} + \frac{d}{dt}\left(\frac{\partial U}{\partial \dot{q}_j}\right)$$

$L = T - U$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0$$

Example: Electromagnetic forces on moving charges

Deduction of lagrangian equation using variational principle for system involving forces not derivable from potential function (non-conservative sys

To include non-conservative forces let us extend the principle assume the form

$$\delta I = \delta \int_{t_1}^{t_2} (T + W)\, dt = 0 \longrightarrow (1)$$

with fixed end points where $\delta w = \delta \sum_i F_i$
$= \sum_i F_i \cdot \delta$

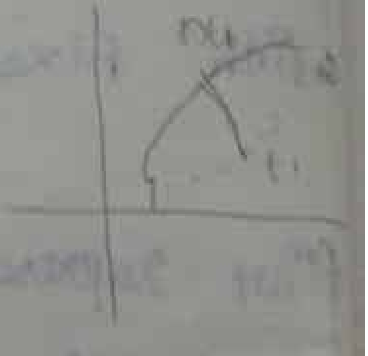
$F_i \delta r_i$ represents the workdone by the force on the system during the virtual displacement from actual to rest or rather varied paths. δ variation is a process that does not include time variation.

Thus the time of motion for the system along every path whether actual or varied is the same. It is possible if we imagine the varied path in configuration space to be built up by a succession of virtual displacements from the actual path of motion.

Thus δq_i refers to virtual displacement of co-ordinates.

∴ The concept of generalised force can used here has indicated in D'Alembert principle.

Possible paths are refers to us $q_j(t, \alpha)$.



$$r_i = r_i[q_j(t,\alpha), t]$$

$$\delta r_i = \sum_j \frac{\partial r_i}{\partial q_j}\delta q_j$$

The component of generalised force expressed as

$$\delta W = \sum_i F_i \cdot \delta r_i$$

$$= \sum_{i,j} F_i \frac{\partial r_i}{\partial q_j} \delta q_j$$

$$= \sum_j Q_j \delta q_j \qquad \because Q_j = \sum_i F_i \frac{\partial r_i}{\partial q_j}$$

equation (1) takes the form,

$$\delta \int_1^2 T\,dt + \delta \int_1^2 W\,dt = 0$$

$$\delta \int_1^2 T\,dt + \int_1^2 \sum_j Q_j \delta q_j\,dt = 0 \longrightarrow (2)$$

Since T is a function of $(q_j, \dot{q}_j$

the First term above expression has Follows.

$$\delta \int_1^2 T(q_j, \dot{q}_j)\,dt$$

$$= \int_1^2 \sum_j \left(\frac{\partial T}{\partial q_j} \delta q_j + \frac{\partial T}{\partial \dot{q}_j} \delta \dot{q}_j \right) dt$$

$$= \int_1^2 \sum_j \left(\frac{\partial T}{\partial q_j} \delta q_j \right) dt + \int_1^2 \sum_j \underline{\frac{\partial T}{\partial \dot{q}_j}}\, \underline{\delta \dot{q}_j}\, dt$$

Integrating by the path second term

$$= \int_1^2 \sum_j \left(\frac{\partial T}{\partial q_j} \delta q_j \right) dt + \sum_j \frac{\partial T}{\partial \dot{q}_j} \delta q_j - \int_1^2 \delta q_j \frac{d}{dt} \ldots$$

The middle term is zero ha[...] it is a variation is fixed end point. Therefore the expression

$$= \int_1^2 \sum_j \frac{\partial T}{\partial q_j} \delta q_j \, dt - \int_1^2 \sum_j \delta q_j \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) dt =$$

$$= \int_1^2 \sum_j \left[\frac{\partial T}{\partial q_j} - \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)\right] \delta q_j \, dt$$

Now eqn (2)

$$= \int_1^2 \sum_j \left[\frac{dT}{dq_j} - \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)\right] \delta q_j \, dt + \int_1^2 Q_j \cdot \delta q_j \, dt$$

$$= \int_1^2 \sum_j \left[\frac{\partial T}{\partial q_j} - \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) + Q_j \right] \delta q_j \, dt = 0$$

Since the constrains are assum[ed] to be holonomic δq_j are independent of eachother ∴ the term within the brack[et] ze[ro]

Cyclic (or) ignorable co-ordinates

The lagrangian L is a function of the generalised co-ordinate q_j, generalised velocity $\dot{q}_j$ and time t and if the lagrangian of the system does not contain a particular co-ordinate q_k. Then $\frac{\partial L}{\partial q_k} = 0$. Such a co-ordinate is refer to a ignorable (or) cyclic co-ordinate. Ex: A particle moving under central force.

Central force is that inwhich a force is always directed towards a fixed centre and the magnitude is a function of distance from the fixed centre. now let us consider of a particle mass is moving in a plane an attracted to origin of co-ordinates with the force that is inversly proportional to square of the distance from it.

If (r, θ) is be plane polar co-ordinates. Then the kinetic energy can written as,

$$T = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\theta}^2)$$

$F \propto \frac{1}{r^2}$

$\Rightarrow F = k \frac{1}{r^2}$

Potential energy, $V = -\int_{\infty}^{r} F \cdot dr$

$$V = -\int_{\infty}^{r} -\frac{k}{r^2} \, dr$$

$$V = -\frac{k}{r}$$

Since the force F is attractive in nature varies inversly as the square of distance from the origin $F = -\frac{k}{r^2}$

Lagrangian $L = T - V$

$$L = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\theta}^2) + \frac{k}{r}$$

The lagrangian equ of motion in this case can be written as,

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j}$

$q_j = r$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right) - \frac{\partial L}{\partial r} = 0$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0$$

[here the lagrangian L is a function of $(r, \theta, \dot{\theta})$

but the lagrangian L does not contain θ. $\therefore \frac{\partial L}{\partial \theta} = 0$ hence θ is called cyclic (or) ignorable co-ordinates]

$$L = \frac{1}{2} m(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{k}{r}$$

$$\frac{\partial L}{\partial r} = \frac{1}{2} m(2r\dot{\theta}^2) - \frac{k}{r^2} = mr\dot{\theta}^2 - \frac{k}{r^2}$$

$$\frac{\partial L}{\partial \dot{r}} = \frac{1}{2} m(2\dot{r}) = m\dot{r}$$

$$\frac{\partial L}{\partial \dot{\theta}} = \frac{1}{2} m\, 2r^2\dot{\theta} = mr^2\dot{\theta}$$

$$\frac{\partial L}{\partial \theta} = 0$$

$$\frac{d}{dt}(m\dot{r}) - mr\dot{\theta}^2 + \frac{k}{r^2} = 0$$

$$m\ddot{r} - mr\dot{\theta}^2 + \frac{k}{r^2} = 0 \longrightarrow (A)$$

$$\frac{d}{dt}(mr^2\dot{\theta}) - 0 = 0$$

$$\cancel{mr^2\ddot{\theta} = 0}$$

$$= m\left(r^2\ddot{\theta} + \dot{\theta}\, 2r\frac{dr}{dt}\right)$$

$$= mr^2\ddot{\theta} + 2mr\dot{\theta}\dot{r}$$

$$= 2mr\dot{r}\dot{\theta} + mr^2\ddot{\theta}$$

$$2mr\dot{r}\dot{\theta} + mr^2\ddot{\theta} = 0 \longrightarrow (5)$$

If A and B describe equn of motion moving under the influence of a central force.

Generalised momentum

It is also termed as Conjucate or canonical momentum. Let us consider a system of mass points acted upon by forces derived from potentials dependent on position only. Then it is conservative force for a such a sys called conservative one. the lagrangian equation of motion can be written as,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0$$

q_j is cyclic: That is L does not contain particular co-ordinate. Then, $\frac{\partial L}{\partial q_j} = 0$

∴ The lagrange's equ becomes

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) = 0$$

(ie) $\frac{\partial L}{\partial \dot{q}_j}$ = a constant = p_j (generalised momentum)

p_j = constant implies that the generalised momentum ① conjucate to a cyclic co-ordinate is conserved.

Routh's procedure

It is a procedure to elimina the conservation ignorable or cyclic co-ordi. If we consider the lagrangian for a syst under central force (Keplers problem)

$$L = \frac{1}{2} m\dot{r}^2 + \frac{1}{2} r^2\dot{\theta}^2 + \frac{k}{r}$$

where k is force constant. r and θ are generalised coordinates. here θ is cyclic

$$\therefore P_\theta = \frac{\partial L}{\partial \dot{\theta}} = mr^2\dot{\theta} = \text{constant}$$

∴ angular momentum is conserved

eventhough θ is cyclic. The lagrangian formulation gives information about how θ varies with time. hence we continued to considered θ but there is a procedure discussed below which eliminates the necessity of this conservation. which is called Routh's procedure.

here we write function R called Routhing function which does not contain generalised velocities corresponding to the ignorable co-ordinate.

The lagrangian L is a function of q_j

$$L = L(q_1 \ldots q_n, \dot{q}_1, \dot{q}_2 \ldots \dot{q}_n, t)$$

Now, If the co-ordinate $q_1, \ldots q_k$ are ignorable

Then

$$L = L(q_{k+1} \ldots q_n, \dot{q}_1 \ldots \dot{q}_n, t)$$

$$\left(R = R(q_{k+1} \ldots q_n, \dot{q}_{k+1} \ldots \dot{q}_n, t)\right)$$

$$\delta L = \sum_{j=k+1}^{n} \frac{\partial L}{\partial q_j} \delta q_j + \sum_{j=1}^{n} \frac{\partial L}{\partial \dot{q}_j} \delta \dot{q}_j + \frac{\partial L}{\partial t} \delta t$$

$$\delta L - \sum_{j=1}^{k} \frac{\partial L}{\partial \dot{q}_j} \delta \dot{q}_j = \sum_{j=k+1}^{n} \frac{\partial L}{\partial q_j} \delta q_j + \sum_{j=k+1}^{n} \frac{\partial L}{\partial \dot{q}_j} \delta \dot{q}_j + \frac{\partial L}{\partial t} \delta t$$

The Routhen function are in which the velocities $\dot{q}_1 \dots \dot{q}_k$ corresponding to the cyclic co-ordinates $q_1 \dots q_k$ are eliminated

$$R = R(q_{k+1} \dots q_n, \dot{q}_{k+1} \dots \dot{q}_n, t)$$

$$\delta R = \sum_{j=k+1}^{n} \frac{\partial R}{\partial q_j} \delta q_j + \sum_{j=k+1}^{n} \frac{\partial R}{\partial \dot{q}_j} \delta \dot{q}_j + \frac{\partial R}{\partial t} \delta t \rightarrow$$

Let us defined the routhen function R as

$$\boxed{R = L - \sum_{j=1}^{k} \dot{q}_j p_j}$$

$uv = uv' + vu'$

$$\delta R = \delta L - \sum_{j=1}^{k} \dot{q}_j \delta p_j - \sum_{j=1}^{k} p_j \delta \dot{q}_j \quad (p_j =$$

$$\delta R = \delta L - \sum_{j=1}^{k} \frac{\partial L}{\partial \dot{q}_j} \delta \dot{q}_j - \sum_{j=1}^{k} \dot{q}_j \delta p_j$$

$$= \sum_{j=k+1}^{n} \frac{\partial L}{\partial q_j} \delta q_j + \sum_{j=k+1}^{n} \frac{\partial L}{\partial \dot{q}_j} \delta \dot{q}_j + \frac{\partial L}{\partial t} \delta t -$$

$$\sum_{j=1}^{k} \dot{q}_j \delta p_j - \sum_{j=1}^{k} \underbrace{\frac{\partial L}{\partial \dot{q}_j}}_{0} \delta \dot{q}_j$$

$$= \sum_{j=k+1}^{n} \frac{\partial L}{\partial q_j} \delta q_j + \sum_{j=k+1}^{n} \frac{\partial L}{\partial \dot{q}_j} \delta \dot{q}_j - \sum_{j=1}^{k} \dot{q}_j \, \delta p_j + \frac{\partial L}{\partial t} \, dt \quad \text{③}$$

Comparing eqn ② & ③ and equating the co-efficients are varying quantities independent

$$\left. \begin{aligned} \frac{\partial L}{\partial q_j} &= \frac{\partial R}{\partial q_j} \\ \frac{\partial L}{\partial \dot{q}_j} &= \frac{\partial R}{\partial \dot{q}_j} \end{aligned} \right\} \rightarrow \text{④}$$

[but here $j = k+1 \ldots n$]

The lagrangian equation can be written as,

$$\sum_{j=k+1}^{n} \frac{d}{dt}\left(\frac{\partial R}{\partial \dot{q}_j}\right) - \frac{\partial R}{\partial q_j} = 0 \longrightarrow \text{⑤}$$

here the ~~routhen~~ lagrangian function is replaced by lagra routhen function and there are only n-k second order equations in the non ignorable variation. This is how routhen producere we can eliminate ignorable co-ordinate from the equation of motion

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = 0$$

This is the lagrangian eqn of motion fo a non-conservative system.

Deduction of hamilton's variational princip From D'Alembert's principle

D'Alembert's principle is

$$\sum_i (F_i - \dot{p}_i) \cdot \delta r_i = 0$$

$$\sum_i F_i \cdot \delta r_i = \sum_i \dot{p}_i \cdot \delta r_i \longrightarrow (1)$$

Considering

$$\sum_i \dot{p}_i \cdot \delta r_i = m_i \cdot \ddot{r}_i \cdot \delta r_i$$

$$= \frac{d}{dt}\left(m_i \frac{dr_i}{dt}\right) \cdot \delta r_i$$

$$\delta(dx) = d(\delta x)$$

$$\delta\left(\frac{dr_i}{dt}\right) = \frac{d}{dt}(\delta r_i) - \frac{dr_i}{dt}\frac{d}{dt}(\delta r_i)$$

we know that δ con

inpreted and an opeararator which

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