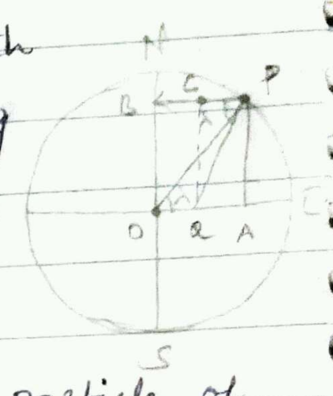
Variation of g with latitude or rotation of the earth:

Let us assume that the earth is a uniform sphere of radius R revolving about its polar diameter NS.

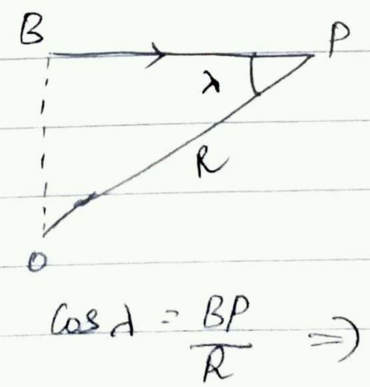


Consider a particle of mass m on the surface of the earth at a latitude λ.

If the earth were at rest, a particle of mass m placed at P will experience a force mg along the radius PO towards O.

Let ω be the angular velocity of the earth. As the earth revolves, the particle at P will execute circular motion with B as centre and BP as radius.

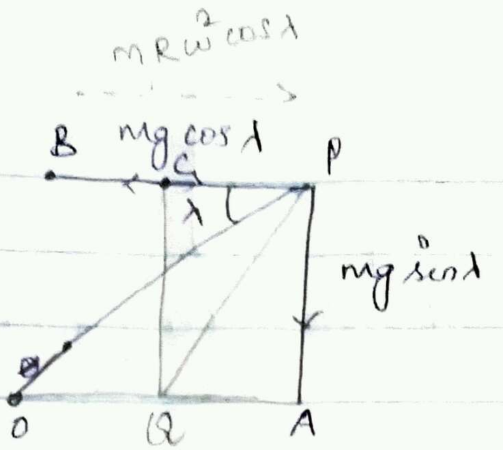
A centrifugal force will develop and the centrifugal force acting on P along BP, away from $B = mBP\omega^2$



$$\cos\lambda = \frac{BP}{R} \Rightarrow BP = \cos\lambda R$$

$$B = m(R\cos\lambda)\omega^2$$
$$= mR\omega^2\cos\lambda$$

Force mg acts along PO. Resolving mg into two Rectangular components



(i) $mg \sin\lambda$ along PA

(ii) $mg \cos\lambda$ along PB.

Let the net force be represented by PC. Then

$$PC = mg\cos\lambda - mR\omega^2\cos\lambda \rightarrow (1)$$

$$PA = mg\sin\lambda \rightarrow (2)$$

The resultant force (mg') experienced by P is along PQ,

$$(PQ)^2 = (PC)^2 + (PA)^2 \text{ (or) } PQ = \left[(PC)^2 + (PA)^2\right]^{1/2}$$

Sub eqn (1) & (2),

$$mg' = \left[\underbrace{\left[mg\cos\lambda - mR\omega^2\cos\lambda\right]^2}_{(a-b)^2} + \left[mg\sin\lambda\right]^2\right]^{1/2}$$

$$mg' = \left[\left(m^2g^2\cos^2\lambda + m^2R^2\omega^4\cos^2\lambda - 2m^2gR\omega^2\cos^2\lambda\right) + \left(m^2g^2\sin^2\lambda\right)\right]^{1/2}$$

$$= (m^2g^2)^{1/2}\left[\cos^2\lambda + \frac{R^2\omega^4}{g^2}\cos^2\lambda - \frac{2R\omega^2}{g}\cos^2\lambda + \sin^2\lambda\right]^{1/2}$$

$$= mg\left[\left(1 + \frac{R^2\omega^4}{g^2}\cos^2\lambda - \frac{2R\omega^2}{g}\cos^2\lambda\right)\right]^{1/2}$$

$$= mg\left[1 - \frac{2R\omega^2}{g}\cos^2\lambda\right]^{1/2}$$ ($\because$ neglecting higher terms $R^2\omega^4\cos^2\lambda$)

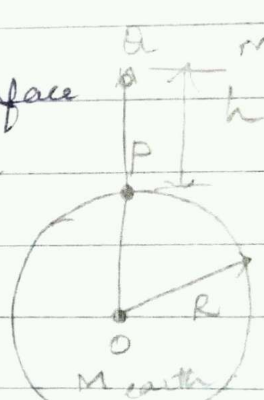
$$mg\left[1-\frac{R\omega^2\cos^2\lambda}{g}\right]$$

$\left(\because \frac{R\omega^2}{g}\right.$ is small, its higher powers can be neglected$\left.\right)$

$$g' = g\left[1-\frac{R\omega^2\cos^2\lambda}{g}\right].$$

[illegible]

Variation of g with altitude:



* Let P be a point on the Surface of the earth and Q another point at an altitude h.

Mass of the earth = M
Radius of the earth = R

Let g be the acceleration due to gravity on the Surface of the earth. Then

$$\left.\begin{array}{l}\text{The force experienced by}\\ \text{a body of mass } m \text{ at } P\end{array}\right\} = mg = \frac{GMm}{R^2} \rightarrow (1)$$

$$\left.\begin{array}{l}\text{The force experienced by}\\ \text{a body of mass } m \text{ at } Q\end{array}\right\} = mg' = \frac{GMm}{(R+h)^2} \rightarrow (2)$$

$g' \rightarrow$ acceleration due to gravity at an altitude h.

Divide (2) by (1)

$$\frac{\cancel{m}g'}{\cancel{m}g} = \frac{\frac{\cancel{G}\cancel{M}\cancel{m}}{(R+h)^2}}{\frac{\cancel{G}\cancel{M}\cancel{m}}{R}}$$

$$\frac{g'}{g} = \frac{R^2}{(R+h)^2} = \frac{R^2}{R^2+h^2+2Rh}$$

$(a+b)^2$

$$= \frac{\cancel{R^2}}{\cancel{R^2}\left(1+\frac{h^2}{R^2}+\frac{2h}{R}\right)}$$

$(1+h/R)^2$

$= 1 + \frac{h^2}{R^2} + \frac{2h}{R}$

$$= \frac{1}{\left[1+(h/R)\right]^2}$$

$$= \left[1+h/R\right]^{-2}$$

$$= 1 - \frac{2h}{R} + \frac{2h^2}{R^2} + \cdots$$

neglecting higher terms

$$\frac{g'}{g} \Rightarrow 1 - \frac{2h}{R}.$$

$$g' = g\left(1 - \frac{2h}{R}\right).$$

This shows that the acceleration due to gravity decreases with increase in altitude.

$g \downarrow \quad h \uparrow$

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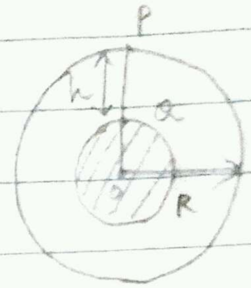
Variation of g with depth:

Let g and g' be the values of acceleration due to gravity at P and Q respectively. At P, the whole mass of the earth attracts the body.



$$mg = \frac{GMm}{R^2} \longrightarrow (1)$$

where m = mass of the body

M = mass of the earth

R = Radius of the earth

At Q, the body is attracted by the mass of the earth of radius $(R-h)$

$$mg' = \frac{GM'm}{(R-h)^2} \longrightarrow (2)$$

Here $M = \frac{4}{3}\pi R^3 \rho$ & $M' = \frac{4}{3}\pi (R-h)^3 \rho$

where $\rho \rightarrow$ mean density of the earth

Dividing (2) by (1)

$$\cancel{\frac{g'}{g}}$$

$$\frac{\cancel{m}g'}{\cancel{m}g} = \frac{\frac{GM'\cancel{m}}{(R-h)^2}}{\frac{GM\cancel{m}}{R^2}}$$

$$\frac{g'}{g} = \frac{M'}{M}\,\frac{R^2}{(R-h)^2}$$

$$= \frac{\cancel{\frac{4}{3}}\cancel{\pi}(R-h)^{\cancel{3}\,1}\cancel{\rho}}{\cancel{\frac{4}{3}}\cancel{\pi}R^{\cancel{3}\,1}\cancel{\rho}} \times \frac{R^{\cancel{2}}}{(R-h)^{\cancel{2}}}$$

$$\frac{g'}{g} = \frac{R-h}{R} = \frac{\cancel{R}}{\cancel{R}} - \frac{h}{R} = 1 - h/R.$$

$$g' = g(1 - h/R)$$

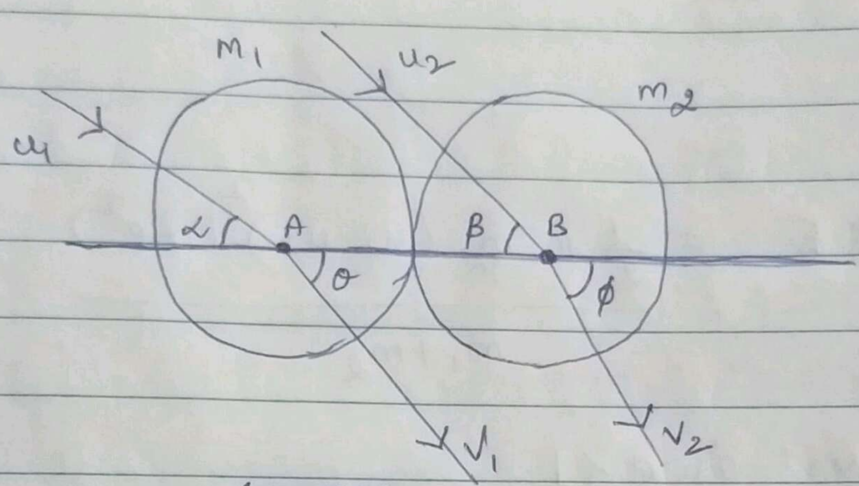
Thus, the acceleration due to gravity decreases with increase of depth.

$g \downarrow$ depth $\uparrow$.

Oblique impact of two smooth Sphere:

A smooth sphere of mass m_1 moving with velocity u_1 impinges obliquely on a smooth sphere of mass m_2 moving with velocity u_2.

If the directions of motion before impact makes angles α and β with common normal, Then velocities and directions of sphere after impact would be,

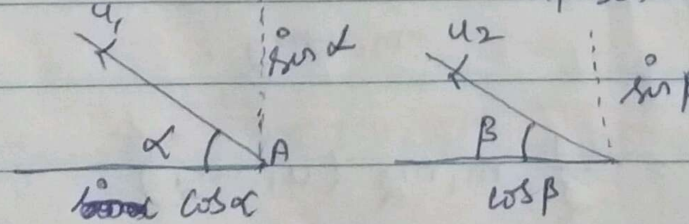


$AB \rightarrow$ common normal

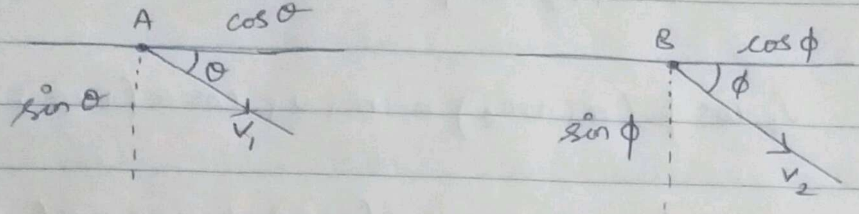
$V_1, V_2 \rightarrow$ velocities of two spheres after impact making angles θ and ϕ with common normal AB.

Before impact velocities: $u_1 \cos\alpha$ and $u_2 \cos\beta$ (along AB)

$u_1 \sin\alpha$ and $u_2 \sin\beta$ (perpendicular to AB)



After impact velocities : $v_1 \cos\theta$ and $v_2 \cos\phi$ (Along AB)
$v_1 \sin\theta$ and $v_2 \cos\phi$ (perpendicular to AB)



Principle of conservation of linear momentum,

$$m_1 v_1 \cos\theta + m_2 v_2 \cos\phi = m_1 u_1 \cos\alpha + m_2 u_2 \cos\beta \rightarrow (1)$$

By newton's experimental law,

$$v_1 \cos\theta - v_2 \cos\phi = -e(u_1 \cos\alpha - u_2 \cos\beta) \rightarrow (2)$$

multiply eqn (2) by m_2

$$m_2 v_1 \cos\theta - m_2 v_2 \cos\phi = -m_2 e(u_1 \cos\alpha - u_2 \cos\beta) \rightarrow (2a)$$

Add (2a) & (1)

$$m_2 v_1 \cos\theta - \cancel{m_2 v_2 \cos\phi} + m_1 v_1 \cos\theta + \cancel{m_2 v_2 \cos\phi} = -m_2 e u_1 \cos\alpha + m_2 e u_2 \cos\beta + m_1 u_1 \cos\alpha + m_2 u_2 \cos\beta$$

$$v_1 \cos\theta (m_1 + m_2) = u_1 \cos\alpha (m_1 - m_2 e) + m_2 u_2 \cos\beta (1+e)$$

$$v_1 \cos\theta = \frac{u_1 \cos\alpha (m_1 - m_2 e) + m_2 u_2 \cos\beta (1+e)}{(m_1 + m_2)}$$

multiply eqn (2) by m_1

$$m_1 v_1 \cos\theta - m_1 v_2 \cos\phi = -m_1 e(u_1 \cos\alpha - u_2 \cos\beta) \rightarrow (2b)$$

subtract ① − (2b)

$$\cancel{m_1 v_1 \cos\theta} + m_2 v_2 \cos\phi - \cancel{m_1 v_1 \cos\theta} + m_1 v_2 \cos\phi = m_1 u_1 \cos\alpha + m_2 u_2 \cos\beta + m_1 e u_1 \cos\alpha - m_1 e u_2 \cos\beta$$

$$v_2 \cos\phi\,(m_1 + m_2) = m_1 u_1 \cos\alpha\,(1+e) + u_2 \cos\beta\,(m_2 - m_1 e)$$

$$v_2 \cos\phi = \frac{m_1 u_1 \cos\alpha\,(1+e) + u_2 \cos\beta\,(m_2 - m_1 e)}{m_1 + m_2}.$$

Special cases:

(i) The impulse of the blow on the sphere of mass m_1 = its change of momentum measured along common normal.

$$m_1 v_1 \cos\theta - m_1 u_1 \cos\alpha = m_1 (v_1 \cos\theta - u_1 \cos\alpha)$$

$$= m_1 \left[\frac{u_1 \cos\alpha\,(m_1 - m_2 e) + m_2 u_2 \cos\beta\,(1+e)}{m_1 + m_2} - u_1 \cos\alpha \right]$$

$$= m_1 \left[\frac{\cancel{u_1 \cos\alpha\, m_1} - u_2 m_2 e \cos\alpha + m_2 u_2 \cos\beta + m_2 u_2 e \cos\beta - \cancel{u_1 \cos\alpha\, m_1} - u_1 \cos\alpha\, m_2}{m_1 + m_2} \right]$$

$$= \frac{-m_1 u_2 m_2 e \cos\alpha + m_1 m_2 u_2 \cos\beta + m_1 m_2 u_2 e \cos\beta - m_1 u_1 \cos\alpha\, m_2}{m_1 + m_2}$$

$$= \frac{m_1 m_2 (1+e)(u_2 \cos\beta - u_1 \cos\alpha)}{m_1 + m_2}.$$

Loss of K.E due to oblique impact:

$$= \frac{1}{2} \frac{m_1 m_2 (1-e^2)}{m_1+m_2} (u_1 \cos\alpha - u_2 \cos\beta)^2$$

Velocities of spheres perpendicular to common normal are unaltered.

Loss of K.E due to Direct impact of two smooth Sphere:

Conservation of linear momentum,

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2 \longrightarrow (1)$$

Newton's experimental law,

$$v_1 - v_2 = -e(u_1 - u_2) \longrightarrow (2)$$

Square both equation

$$(m_1 v_1 + m_2 v_2)^2 = (m_1 u_1 + m_2 u_2)^2$$

$$m_1^2 v_1^2 + m_2^2 v_2^2 + 2m_1 m_2 v_1 v_2 = (m_1 u_1 + m_2 u_2)^2 \longrightarrow (1a)$$

$$(v_1 - v_2)^2 = (-e)^2 (u_1 - u_2)^2$$

$$v_1^2 + v_2^2 - 2v_1 v_2 = e^2 (u_1 - u_2)^2 \longrightarrow (2a)$$

multiplying $m_1 m_2$ with equation (2a)

$$m_1 m_2 v_1^2 + m_1 m_2 v_2^2 - 2m_1 m_2 v_1 v_2 = m_1 m_2 e^2 (u_1 - u_2)^2 \longrightarrow (2b)$$

Adding (1a) & (2b)

$$m_1^2 v_1^2 + m_2^2 v_2^2 + \cancel{2m_1 m_2 v_1 v_2} + m_1 m_2 v_1^2 + m_1 m_2 v_2^2 - \cancel{2m_1 m_2 v_1 v_2} = (m_1 u_1 + m_2 u_2)^2 + m_1 m_2 e^2 (u_1 - u_2)^2$$

$$\underbrace{m_1^2 v_1^2}_{} + \underbrace{m_2^2 v_2^2}_{} + \underbrace{m_1 m_2 v_1^2}_{} + \underbrace{m_1 m_2 v_2^2}_{} = (m_1 u_1 + m_2 u_2)^2 + m_1 m_2 e^2 (u_1 - u_2)^2$$

$$(m_1^2 + m_1 m_2) v_1^2 + (m_2^2 + m_1 m_2) v_2^2$$
$$= (m_1 u_1 + m_2 u_2)^2 + e^2 m_1 m_2 (u_1 - u_2)^2$$

$$m_1 (m_1 + m_2) v_1^2 + m_2 (m_2 + m_1) v_2^2$$
$$= (m_1 u_1 + m_2 u_2)^2 + e^2 m_1 m_2 (u_1 - u_2)^2$$

$$(m_1 + m_2)(m_1 v_1^2 + m_2 v_2^2) = (m_1 u_1 + m_2 u_2)^2 + e^2 m_1 m_2 (u_1 - u_2)^2$$

Add and subtract $m_1 m_2 (u_1 - u_2)^2$ in R. H. S

$$(m_1 + m_2)(m_1 v_1^2 + m_2 v_2^2) = (m_1 u_1 + m_2 u_2)^2 + m_1 m_2 (u_1 - u_2)^2$$
$$+ e^2 m_1 m_2 (u_1 - u_2)^2 - m_1 m_2 (u_1 - u_2)^2$$

$$= m_1^2 u_1^2 + m_2 u_2^2 + 2 m_1 m_2 u_1 u_2$$
$$+ m_1 m_2 (u_1^2 + u_2^2 - 2 u_1 u_2)$$
$$+ e^2 m_1 m_2 (u_1 - u_2)^2 - m_1 m_2 (u_1 - u_2)^2$$

$$= \underline{m_1^2 u_1^2} + \underline{m_2^2 u_2^2} + \cancel{2 m_1 m_2 u_1 u_2}$$
$$+ \underline{m_1 m_2 u_1^2} + \underline{m_1 m_2 u_2^2} - \cancel{2 m_1 m_2 u_1 u_2}$$
$$+ \underline{e^2 m_1 m_2 (u_1 - u_2)^2} - \underline{m_1 m_2 (u_1 - u_2)^2}$$

$$= m_1 (m_1 u_1^2 + m_2 u_2^2) + m_2 (m_2 u_2^2 + m_1 u_1^2)$$
$$+ (e^2 - 1) m_1 m_2 (u_1 - u_2)^2$$

$$(m_1 + m_2)(m_1 v_1^2 + m_2 v_2^2) = (m_1 + m_2)(m_1 u_1^2 + m_2 u_2^2) - (1 - e^2) m_1 m_2 (u_1 - u_2)^2$$

$$m_1 v_1^2 + m_2 v_2^2 = \frac{(m_1 u_1^2 + m_2 u_2^2)(m_1+m_2) - m_1 m_2 (u_1-u_2)^2 (1-e^2)}{(m_1+m_2)}$$

$$m_1 v_1^2 + m_2 v_2^2 = \frac{(m_1 u_1^2 + m_2 u_2^2)\cancel{(m_1+m_2)}}{\cancel{(m_1+m_2)}} - \frac{m_1 m_2 (u_1-u_2)^2 (1-e^2)}{m_1+m_2}$$

multiplying $\frac{1}{2}$ in the above equation.

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 v_2^2 - \boxed{\frac{\frac{1}{2} m_1 m_2 (u_1-u_2)^2 (1-e^2)}{m_1+m_2}}$$

Loss of K.E

$$\text{Loss of K.E} = \frac{\frac{1}{2} m_1 m_2 (u_1-u_2)^2 (1-e^2)}{m_1+m_2}.$$

special cases: (i) $e<1$.

$(1-e^2)$ is positive, $(u_1-u_2)^2$ is always positive

Hence, loss of K.E due to impact, it is then coverted ~~as~~ into sound, heat or vibration.

(ii) $e=0$.

$$K.E = \frac{\frac{1}{2} m_1 m_2 (u_1-u_2)^2}{m_1+m_2} (1-0)$$

$$K.E = \frac{\frac{1}{2} m_1 m_2 (u_1-u_2)^2}{m_1+m_2}.$$

maximum loss of K.E on impact is plastic bodies.